Errata Corrige

Marta Cialdea Mayer

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The example of Figure 1 in the paper "A Proof Procedure for Hybrid Logic with Binders, Transitivity and Relation Hierarchies" (CADE 24) contains a flaw: node 9 is not derivable by means of the Trans rule from nodes 4, 7, 1 and 2: in order to be derivable, the minor premiss should be labelled by $a_1{:}\Diamond_R a_2$ instead of $a_2{:}\Diamond_R a_1$. Consequently, all the nodes deriving from 9 (12,13,14,23,26,29,30,31,38,41) are not in the branch, as well as other nodes because of different blockings.

A complete and open branch for $\{(\mathsf{A}{\downarrow}x.\Diamond_{R^-}\Diamond_R\neg x)\wedge\Box_R p, \mathsf{Trans}(R)\}$ is represented below, where $F=(\mathsf{A}{\downarrow}x.\Diamond_{R^-}\Diamond_R\neg x)\wedge\Box_R p$ and $G=\Diamond_{R^-}\Diamond_R\neg x$ (node numbering and nominal names are left the same as in Figure 1 in the cited paper).

0)	$a_1{:}F$	
1)	$\mathsf{Trans}(R)$	
2)	$R\sqsubseteq R$	
3)	$a_1{:}\mathsf{A}{\downarrow}x.G$	$0\rightsquigarrow^{\wedge} 3$
4)	$a_1{:}\Box_R p$	$0\rightsquigarrow^{\wedge} 4$
5)	$a_1{:}{\downarrow}x.G$	$(3,0)\rightsquigarrow^{\mathsf{A}} 5$
6)	$a_1{:}\Diamond_{R^-}\Diamond_R\neg a_1$	$5\rightsquigarrow^{\downarrow} 6$
7)	$a_2{:}\Diamond_R a_1$	$6\rightsquigarrow^{\Diamond} 7$
8)	$a_2{:}\Diamond_R\neg a_1$	$6\rightsquigarrow^{\Diamond} 8$
10)	$a_2{:}\Diamond_R a_3$	$8\rightsquigarrow^{\Diamond} 10$
11)	$a_3{:}\neg a_1$	$8\rightsquigarrow^{\Diamond} 11$
15)	$a_3{:}{\downarrow}x.G$	$(3,10)\rightsquigarrow^{\mathsf{A}} 15$
16)	$a_2{:}{\downarrow}x.G$	$(3,7)\rightsquigarrow^{\mathsf{A}} 16$
17)	$a_3{:}\Diamond_{R^-}\Diamond_R\neg a_3$	$15\rightsquigarrow^{\downarrow} 17$
18)	$a_2{:}\Diamond_{R^-}\Diamond_R\neg a_2$	$16\rightsquigarrow^{\downarrow} 18$
19)	$a_4{:}\Diamond_R a_3$	$17\rightsquigarrow^{\Diamond} 19$
20)	$a_4{:}\Diamond_R\neg a_3$	$17\rightsquigarrow^{\Diamond} 20$
24)	$a_4{:}\Diamond_R a_6$	$20\rightsquigarrow^{\Diamond} 24$
25)	$a_6{:}\neg a_3$	$20\rightsquigarrow^{\Diamond} 25$
32)	$a_6{:}{\downarrow}x.G$	$(3,24)\rightsquigarrow^{\mathsf{A}} 32$
33)	$a_4{:}{\downarrow}x.G$	$(3,19)\rightsquigarrow^{\mathsf{A}} 33$
34)	$a_6{:}\Diamond_{R^-}\Diamond_R\neg a_6$	$32\rightsquigarrow^{\downarrow} 34$
35)	$a_4{:}\Diamond_{R^-}\Diamond_R\neg a_4$	$33\rightsquigarrow^{\downarrow} 35$

In this branch and all its sub-branches, all non-top nominals are pairwise compatible, therefore nodes 18, 34 and 35 are blocked by 17 and never expanded.

Once corrected, this example does not show at all the dynamic nature of blockings.

A more significant example is given in Figure 1, representing a complete and open tableau branch $\mathcal{B}$ for the assertion $\mathsf{Trans}(R)$ and the formula

$$F=\Diamond_R\top\wedge\mathsf{A}\Box_{R^-}p\wedge\Box_R G \quad\text{where}\quad G={\downarrow}x.\Diamond_R{\downarrow}y.x{:}\Diamond_{R^-}\neg y$$

In the comments below, the notation $\mathcal{B}_n$ is used to denote the branch segment up to node n included. Note that, in this example, the formulae to be taken into account to check compatibilities are p, $\Box_{R^-}p$ and $\Box_R G$.

0)	$a_0\colon F$		22)	$a_2\colon \Diamond_R a_3$	$20 \rightsquigarrow^{\Diamond} 22$
1)	$\mathsf{Trans}(R)$		23)	$a_3\colon {\downarrow} y.a_2\colon \Diamond_R \neg y$	$20 \rightsquigarrow^{\Diamond} 23$
2)	$R \sqsubseteq R$		24)	$a_1\colon \Diamond_R a_4$	$21 \rightsquigarrow^{\Diamond} 24$
3)	$a_0\colon (\Diamond_R \top \wedge \mathsf{A}\Box_{R^-} p)$	$0 \rightsquigarrow^{\wedge} 3$	25)	$a_4\colon \neg a_2$	$21 \rightsquigarrow^{\Diamond} 25$
4)	$a_0\colon \Box_R G$	$0 \rightsquigarrow^{\wedge} 4$	26)	$a_3\colon \Box_R G$	$(17, 22, 1, 2) \rightsquigarrow^{\mathsf{Trans}} 26$
5)	$a_0\colon \Diamond_R \top$	$3 \rightsquigarrow^{\wedge} 5$	27)	$a_3\colon G$	$(17, 22) \rightsquigarrow^{\Box} 27$
6)	$a_0\colon \mathsf{A}\Box_{R^-} p$	$3 \rightsquigarrow^{\wedge} 6$	28)	$a_3\colon a_2\colon \Diamond_R \neg a_3$	$23 \rightsquigarrow^{\downarrow} 28$
7)	$a_0\colon \Diamond_R a_1$	$5 \rightsquigarrow^{\Diamond} 7$	29)	$a_4\colon \Box_R G$	$(11, 24, 1, 2) \rightsquigarrow^{\mathsf{Trans}} 29$
8)	$a_1\colon \top$	$5 \rightsquigarrow^{\Diamond} 8$	30)	$a_4\colon G$	$(11, 24) \rightsquigarrow^{\Box} 30$
9)	$a_1\colon \Box_{R^-} p$	$(6, 7) \rightsquigarrow^{\mathsf{A}} 9$	31)	$a_2\colon \Diamond_R \neg a_3$	$28 \rightsquigarrow^{@} 31$
10)	$a_0\colon \Box_{R^-} p$	$(6, 0) \rightsquigarrow^{\mathsf{A}} 10$	32)	$a_4\colon \Diamond_R {\downarrow} y.a_4\colon \Diamond_R \neg y$	$30 \rightsquigarrow^{\downarrow} 32$
11)	$a_1\colon \Box_R G$	$(4, 7, 1, 2) \rightsquigarrow^{\mathsf{Trans}} 11$	33)	$a_2\colon \Diamond_R a_5$	$31 \rightsquigarrow^{\Diamond} 33$
12)	$a_1\colon G$	$(4, 7) \rightsquigarrow^{\Box} 12$	34)	$a_5\colon \neg a_3$	$31 \rightsquigarrow^{\Diamond} 34$
13)	$a_0\colon p$	$(9, 7) \rightsquigarrow^{\Box} 13$	35)	$a_4\colon \Box_{R^-} p$	$(6, 24) \rightsquigarrow^{\mathsf{A}} 35$
14)	$a_1\colon \Diamond_R {\downarrow} y.a_1\colon \Diamond_R \neg y$	$12 \rightsquigarrow^{\downarrow} 14$	36)	$a_3\colon \Box_{R^-} p$	$(6, 22) \rightsquigarrow^{\mathsf{A}} 36$
15)	$a_1\colon \Diamond_R a_2$	$14 \rightsquigarrow^{\Diamond} 15$	37)	$a_2\colon \Box_{R^-} p$	$(6, 15) \rightsquigarrow^{\mathsf{A}} 37$
16)	$a_2\colon {\downarrow} y.a_1\colon \Diamond_R \neg y$	$14 \rightsquigarrow^{\Diamond} 16$	38)	$a_1\colon p$	$(35, 24) \rightsquigarrow^{\Box} 38$
17)	$a_2\colon \Box_R G$	$(11, 15, 1, 2) \rightsquigarrow^{\mathsf{Trans}} 17$	39)	$a_2\colon p$	$(36, 22) \rightsquigarrow^{\Box} 39$
18)	$a_2\colon G$	$(11, 15) \rightsquigarrow^{\Box} 18$	40)	$a_4\colon \Diamond_R a_6$	$32 \rightsquigarrow^{\Diamond} 40$
19)	$a_2\colon a_1\colon \Diamond_R \neg a_2$	$16 \rightsquigarrow^{\downarrow} 19$	41)	$a_6\colon {\downarrow} y.a_4\colon \Diamond_R \neg y$	$32 \rightsquigarrow^{\Diamond} 41$
20)	$a_2\colon \Diamond_R {\downarrow} y.a_2\colon \Diamond_R \neg y$	$18 \rightsquigarrow^{\downarrow} 20$	42)	$a_6\colon \Box_{R^-} p$	$(6, 40) \rightsquigarrow^{\mathsf{A}} 42$
21)	$a_1\colon \Diamond_R \neg a_2$	$19 \rightsquigarrow^{@} 21$	43)	$a_4\colon p$	$(42, 40) \rightsquigarrow^{\Box} 43$

Figure 1: A complete tableau branch for $\{\Diamond_R \top \wedge \mathsf{A}\Box_{R^-} p \wedge \Box_R G, \mathsf{Trans}(r)\}$, where $G = {\downarrow} x.\Diamond_R {\downarrow} y.x\colon \Diamond_R \neg y$.

The root nodes are (beyond nodes labelled by assertions): 0–6 and 10, and the offspring relation is:

$5 \prec_{\mathcal{B}} \{7 - 9, 11 - 14\}$ $\quad$ $14 \prec_{\mathcal{B}} \{15 - 21, 37\}$

$20 \prec_{\mathcal{B}} \{22, 23, 26 - 28, 31, 36, 39\}$ $\quad$ $21 \prec_{\mathcal{B}} \{24, 25, 29, 30, 32, 35, 38\}$

$31 \prec_{\mathcal{B}} \{33, 34\}$ $\quad$ $32 \prec_{\mathcal{B}} \{40 - 43\}$

For instance, node 7 is the minor premiss of the application of the Trans rule producing 11, and it is also the minor premiss of the application of the $\Box$ rule producing 12 and 13; therefore 7, 11, 12 and 13 are siblings. Moreover, 7 is also the first non-phantom node where a_1 occurs when the A rule is applied to produce node 9 focusing on a_1, therefore 7 is the minor premiss of the inference, thus one of 9's siblings.

As a further example, though node 22 is a phantom in the final branch, it is not a phantom in $\mathcal{B}_{35}$ (see below). The branch $\mathcal{B}_{35}$ is expanded by an application of the A rule focusing on a_3 and producing node 36. In this branch, 22 is the first non-phantom node where a_3 occurs, so it is the minor premiss of the A inference and 22 and 36 are siblings (in all branch segments from $\mathcal{B}_{36}$ onwards).

In the whole branch $\mathcal{B} = \mathcal{B}_{43}$, the nodes 20 and 32 are blocked by 14, because a_1 is compatible with both a_2 and a_4: the relevant formulae such nominals label in the final branch are p, $\Box_{R^-} p$ and $\Box_R G$.

The fact that 20 and 32 are blocked by 14 intuitively means that a_2 and a_4 behave "like" a_1, However, though a_2 and a_4 are compatible, the presence of node 25 does not allow to identify the states they denote in a model of this open branch.

Being 20 and 32 directly blocked in $\mathcal{B}$, all their descendants ($22, 23, 26$–$28, 31, 33, 34, 36, 39$–43) are phantom nodes in $\mathcal{B}$.

However, node 20 is blocked by 14 only in $\mathcal{B}_{37}$ (where a_1 and a_2 label $\Box_{R^-}p$ and $\Box_R G$) and from $\mathcal{B}_{39}$ onwards, when both (38) $a_1\!:p$ and (39) $a_2\!:p$ are added. In particular, 20 is not blocked in $\mathcal{B}_i$ for $i \leq 36$, therefore, it is expanded, and its descendants can also be expanded (or used as minor premisses) till node 39 is added to the branch.

Analogously, 32 is blocked by 14 in $\mathcal{B}_i$ only for $35 \leq i \leq 37$ and $i = 43$. Therefore, for instance, node 40 is not a phantom in $\mathcal{B}_{42}$, so that it can be used as the minor premiss of the application of the $\Box$ rule producing 43. Note also that a_2 and a_4 are compatible in $\mathcal{B}_i$ for $32 \leq i \leq 34$ and $i = 38$ and 20 is not blocked in these branch segments, therefore it blocks 32 (though 20 is not an ancestor of 32 w.r.t. the offspring relation).

In order for node 31 to be blocked by 21, a_1, a_2 and a_3 must be compatible. But when a_1 and a_2 are compatible, node 20 is blocked, and in such a case 31, that is one of 20's children, is a phantom. Therefore 31 is never directly blocked.

The branch is complete: no further expansion are possible without violating the restrictions on blocked nodes. In particular, in the whole branch:

- the A rule cannot focus on a_5, which only occurs in phantom nodes.
- Though nodes 36 and 42, obtained by applications of the A rule, are phantoms, such a rule cannot focus again on a_3 and a_6, which only occur in phantom nodes.
- Though 26 and 27 are phantoms, the Trans and $\Box$ rules cannot use again 22 as a minor premiss, since it is a phantom too.
- Similarly, the other phantom nodes labelled by relational formulae cannot be used as minor premisses. For instance, 40 cannot be used as the minor premiss of an application of the $\Box$ rule, paired with 29.