A Proof Procedure for Hybrid Logic with Binders, Transitivity and Relation Hierarchies (CADE 24)

The calculus in action

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Conventions

- Red node labels: nodes expanded to produce the lastly added node(s).
- Blue node labels: nodes relevant to determine compatibilities.
- Boxed node labels: blockable nodes.
- Orange node labels: blocked nodes.
- Green node labels: blocking nodes.
- Gray node labels: phantom nodes.
- Light blue node labels: phantom nodes relevant to determine compatibilities.
- Pink node labels: phantom nodes expanded to produce the lastly added node(s). This may happen either when the expanded node's label has the form $a\colon \Box_R F$, or when it is the addition of the new node(s) to make the expanded one a phantom.

Example 1

Initial formula: $\Diamond_S \Diamond_S p \wedge \Box_S \neg p$
Assertions: $\{\text{Trans}(R),\ R \sqsubseteq S,\ S \sqsubseteq R\}$

I have an S-successor with an S-successor satisfying p,
None of my S-successors satisfies p
R is transitive and the same as S

- Root nodes: 0–5

Initial tableau:

(0)	$a_0 : F$	
(1)	Trans(R)	
(2)	$R \sqsubseteq S$	
(3)	$S \sqsubseteq R$	
(4)	$R \sqsubseteq R$	Rel_0
(5)	$S \sqsubseteq S$	Rel_0

- Root nodes: 0–5,6,7

(0)	$a_0 \colon F$	
(1)	Trans(R)	
(2)	$R \sqsubseteq S$	
(3)	$S \sqsubseteq R$	
(4)	$R \sqsubseteq R$	Rel_0
(5)	$S \sqsubseteq S$	Rel_0
(6)	$a \colon \Diamond_S \Diamond_S p$	$0 \leadsto^{\wedge} 6$
(7)	$a \colon \Box_S \neg p$	$0 \leadsto^{\wedge} 7$

- Root nodes: 0–5,6,7
- $6 \prec_{\mathcal{B}} \{8, 9\}$

(0)	$a_0 \colon F$	
(1)	Trans(R)	
(2)	$R \sqsubseteq S$	
(3)	$S \sqsubseteq R$	
(4)	$R \sqsubseteq R$	Rel_0
(5)	$S \sqsubseteq S$	Rel_0
(6)	$a \colon \Diamond_S \Diamond_S p$	$0 \rightsquigarrow^{\wedge} 6$
(7)	$a \colon \Box_S \neg p$	$0 \rightsquigarrow^{\wedge} 7$

(8)	$a \colon \Diamond_S b$	$6 \rightsquigarrow^{\Diamond} 8$
(9)	$b \colon \Diamond_S p$	$6 \rightsquigarrow^{\Diamond} 9$

$F = \Diamond_S \Diamond_S p \wedge \Box_S \neg p$, Trans($R$), $R \sqsubseteq S$, $S \sqsubseteq R$

- Root nodes: 0–5,6,7
- $6 \prec_{\mathcal{B}} \{8, 9\}$
- $9 \prec_{\mathcal{B}} \{10, 11\}$

(0)	$a_0 \colon F$	
(1)	Trans(R)	
(2)	$R \sqsubseteq S$	
(3)	$S \sqsubseteq R$	
(4)	$R \sqsubseteq R$	Rel_0
(5)	$S \sqsubseteq S$	Rel_0
(6)	$a \colon \Diamond_S \Diamond_S p$	$0 \leadsto^{\wedge} 6$
(7)	$a \colon \Box_S \neg p$	$0 \leadsto^{\wedge} 7$

(8)	$a \colon \Diamond_S b$	$6 \leadsto^{\Diamond} 8$
(9)	$b \colon \Diamond_S p$	$6 \leadsto^{\Diamond} 9$
(10)	$b \colon \Diamond_S c$	$9 \leadsto^{\Diamond} 10$
(11)	$c \colon p$	$9 \leadsto^{\Diamond} 11$

$F = \Diamond_S \Diamond_S p \wedge \Box_S \neg p$, Trans($R$), $R \sqsubseteq S$, $S \sqsubseteq R$

- Root nodes: 0–5,6,7
- $6 \prec_{\mathcal{B}} \{8, 9, 12\}$
- $9 \prec_{\mathcal{B}} \{10, 11\}$

(0)	$a_0 \colon F$	
(1)	Trans(R)	
(2)	$R \sqsubseteq S$	
(3)	$S \sqsubseteq R$	
(4)	$R \sqsubseteq R$	Rel$_0$
(5)	$S \sqsubseteq S$	Rel$_0$
(6)	$a \colon \Diamond_S \Diamond_S p$	$0 \leadsto^{\wedge} 6$
(7)	$a \colon \Box_S \neg p$	$0 \leadsto^{\wedge} 7$

(8)	$a \colon \Diamond_S b$	$6 \leadsto^{\Diamond} 8$
(9)	$b \colon \Diamond_S p$	$6 \leadsto^{\Diamond} 9$
(10)	$b \colon \Diamond_S c$	$9 \leadsto^{\Diamond} 10$
(11)	$c \colon p$	$9 \leadsto^{\Diamond} 11$
(12)	$a \colon \Diamond_R b$	$(8, 3) \leadsto^{\text{Link}} 12$

- Root nodes: 0–5,6,7
- $6 \prec_{\mathcal{B}} \{8, 9, 12\}$
- $9 \prec_{\mathcal{B}} \{10, 11, 13\}$

(0)	$a_0 \colon F$	
(1)	Trans(R)	
(2)	$R \sqsubseteq S$	
(3)	$S \sqsubseteq R$	
(4)	$R \sqsubseteq R$	Rel_0
(5)	$S \sqsubseteq S$	Rel_0
(6)	$a \colon \Diamond_S \Diamond_S p$	$0 \leadsto^{\wedge} 6$
(7)	$a \colon \Box_S \neg p$	$0 \leadsto^{\wedge} 7$

(8)	$a \colon \Diamond_S b$	$6 \leadsto^{\Diamond} 8$
(9)	$b \colon \Diamond_S p$	$6 \leadsto^{\Diamond} 9$
(10)	$b \colon \Diamond_S c$	$9 \leadsto^{\Diamond} 10$
(11)	$c \colon p$	$9 \leadsto^{\Diamond} 11$
(12)	$a \colon \Diamond_R b$	$(8, 3) \leadsto^{\mathrm{Link}} 12$
(13)	$b \colon \Diamond_R c$	$(10, 3) \leadsto^{\mathrm{Link}} 13$

$F = \Diamond_S \Diamond_S p \wedge \Box_S \neg p$, Trans($R$), $R \sqsubseteq S$, $S \sqsubseteq R$

- Root nodes: 0–5,6,7
- $6 \prec_{\mathcal{B}} \{8, 9, 12, 14\}$
- $9 \prec_{\mathcal{B}} \{10, 11, 13\}$

(0)	$a_0 \colon F$	
(1)	Trans(R)	
(2)	$R \sqsubseteq S$	
(3)	$S \sqsubseteq R$	
(4)	$R \sqsubseteq R$	Rel_0
(5)	$S \sqsubseteq S$	Rel_0
(6)	$a \colon \Diamond_S \Diamond_S p$	$0 \rightsquigarrow^{\wedge} 6$
(7)	$a \colon \Box_S \neg p$	$0 \rightsquigarrow^{\wedge} 7$

(8)	$a \colon \Diamond_S b$	$6 \rightsquigarrow^{\Diamond} 8$
(9)	$b \colon \Diamond_S p$	$6 \rightsquigarrow^{\Diamond} 9$
(10)	$b \colon \Diamond_S c$	$9 \rightsquigarrow^{\Diamond} 10$
(11)	$c \colon p$	$9 \rightsquigarrow^{\Diamond} 11$
(12)	$a \colon \Diamond_R b$	$(8, 3) \rightsquigarrow^{\mathrm{Link}} 12$
(13)	$b \colon \Diamond_R c$	$(10, 3) \rightsquigarrow^{\mathrm{Link}} 13$
(14)	$b \colon \Box_R \neg p$	$(7, 12, 1, 2) \rightsquigarrow^{\mathrm{Trans}} 14$

$F = \Diamond_S \Diamond_S p \wedge \Box_S \neg p$, Trans($R$), $R \sqsubseteq S$, $S \sqsubseteq R$

- Root nodes: 0–5,6,7
- $6 \prec_{\mathcal{B}} \{8, 9, 12, 14, 15\}$
- $9 \prec_{\mathcal{B}} \{10, 11, 13\}$

(0)	$a_0 : F$	
(1)	Trans(R)	
(2)	$R \sqsubseteq S$	
(3)	$S \sqsubseteq R$	
(4)	$R \sqsubseteq R$	Rel_0
(5)	$S \sqsubseteq S$	Rel_0
(6)	$a : \Diamond_S \Diamond_S p$	$0 \leadsto^{\wedge} 6$
(7)	$a : \Box_S \neg p$	$0 \leadsto^{\wedge} 7$

(8)	$a : \Diamond_S b$	$6 \leadsto^{\Diamond} 8$
(9)	$b : \Diamond_S p$	$6 \leadsto^{\Diamond} 9$
(10)	$b : \Diamond_S c$	$9 \leadsto^{\Diamond} 10$
(11)	$c : p$	$9 \leadsto^{\Diamond} 11$
(12)	$a : \Diamond_R b$	$(8, 3) \leadsto^{\mathrm{Link}} 12$
(13)	$b : \Diamond_R c$	$(10, 3) \leadsto^{\mathrm{Link}} 13$
(14)	$b : \Box_R \neg p$	$(7, 12, 1, 2) \leadsto^{\mathrm{Trans}} 14$
(15)	$c : \neg p$	$(14, 13) \leadsto^{\Box} 15$

- Root nodes: 0–5,6,7
- $6 \prec_{\mathcal{B}} \{8, 9, 12, 14, 15\}$
- $9 \prec_{\mathcal{B}} \{10, 11, 13\}$

(0)	$a_0 \colon F$	
(1)	Trans(R)	
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(6)	$a \colon \Diamond_S \Diamond_S p$	$0 \leadsto^{\wedge} 6$
(7)	$a \colon \Box_S \neg p$	$0 \leadsto^{\wedge} 7$

(8)	$a \colon \Diamond_S b$	$6 \leadsto^{\Diamond} 8$
(9)	$b \colon \Diamond_S p$	$6 \leadsto^{\Diamond} 9$
(10)	$b \colon \Diamond_S c$	$9 \leadsto^{\Diamond} 10$
(11)	$c \colon p$	$9 \leadsto^{\Diamond} 11$
(12)	$a \colon \Diamond_R b$	$(8, 3) \leadsto^{\text{Link}} 12$
(13)	$b \colon \Diamond_R c$	$(10, 3) \leadsto^{\text{Link}} 13$
(14)	$b \colon \Box_R \neg p$	$(7, 12, 1, 2) \leadsto^{\text{Trans}} 14$
(15)	$c \colon \neg p$	$(14, 13) \leadsto^{\Box} 15$

The branch is closed

Example 2

Initial formula: $\Diamond_R \top \wedge \mathsf{A}\Box_{R^-} p \wedge \Box_R {\downarrow} x.\Diamond_R {\downarrow} y.x : \Diamond_R \neg y$
Assertions: $\{\mathrm{Trans}(R)\}$

I have at least an R-successor,
The R-predecessor of any state of the model satisfies p,
All my R-descendants have at least two different R-successors.

$F = \Diamond_R \top \wedge \mathsf{A}\Box_{R^-} p \wedge \Box_R G$, where $G = {\downarrow} x.\Diamond_R {\downarrow} y.x{:}\, \Diamond_R \neg y$ and R is transitive

Initial tableau

0) $a_0 : F$

1) $\mathrm{Trans}(R)$

2) $R \sqsubseteq R$

$F = \Diamond_R \top \wedge \mathsf{A}\Box_{R^-} p \wedge \Box_R G$, where $G = {\downarrow}x.\Diamond_R{\downarrow}y.x{:}\,\Diamond_R \neg y$ and R is transitive

0)	$a_0 : F$	
1)	Trans(R)	
2)	$R \sqsubseteq R$	
3)	$a_0 : (\Diamond_R \top \wedge \mathsf{A}\Box_{R^-} p)$	$0 \rightsquigarrow^{\wedge} 3$
4)	$a_0 : \Box_R G$	$0 \rightsquigarrow^{\wedge} 4$

$F = \Diamond_R \top \wedge \mathsf{A}\Box_{R-}\, p \wedge \Box_R G$, where $G = \downarrow x.\Diamond_R \downarrow y.x\colon \Diamond_R \neg y$ and R is transitive

0)	$a_0 \colon F$	
1)	$\mathrm{Trans}(R)$	
2)	$R \sqsubseteq R$	
3)	$a_0 \colon (\Diamond_R \top \wedge \mathsf{A}\Box_{R-}\, p)$	$0 \leadsto^{\wedge} 3$
4)	$a_0 \colon \Box_R G$	$0 \leadsto^{\wedge} 4$
5)	$\boxed{a_0 \colon \Diamond_R \top}$	$3 \leadsto^{\wedge} 5$
6)	$a_0 \colon \mathsf{A}\Box_{R-}\, p$	$3 \leadsto^{\wedge} 6$

$F = \Diamond_R \top \wedge \mathsf{A}\Box_{R^-} p \wedge \Box_R G$, where $G = \downarrow x.\Diamond_R \downarrow y.x{:}\,\Diamond_R \neg y$ and R is transitive

0)	$a_0 : F$	
1)	$\mathrm{Trans}(R)$	
2)	$R \sqsubseteq R$	
3)	$a_0 : (\Diamond_R \top \wedge \mathsf{A}\Box_{R^-} p)$	$0 \leadsto^{\wedge} 3$
4)	$a_0 : \Box_R G$	$0 \leadsto^{\wedge} 4$
5)	$\boxed{a_0 : \Diamond_R \top}$	$3 \leadsto^{\wedge} 5$
6)	$a_0 : \mathsf{A}\Box_{R^-} p$	$3 \leadsto^{\wedge} 6$
7)	$a_0 : \Diamond_R a_1$	$5 \leadsto^{\Diamond} 7$
8)	$a_1 : \top$	$5 \leadsto^{\Diamond} 8$

$F = \Diamond_R \top \wedge \mathsf{A}\Box_{R-} p \wedge \Box_R G$, where $G = {\downarrow}x.\Diamond_R{\downarrow}y.x{:}\, \Diamond_R \neg y$ and R is transitive

0)	$a_0{:}\, F$	
1)	$\mathrm{Trans}(R)$	
2)	$R \sqsubseteq R$	
3)	$a_0{:}\, (\Diamond_R \top \wedge \mathsf{A}\Box_{R-} p)$	$0 \rightsquigarrow^{\wedge} 3$
4)	$a_0{:}\, \Box_R G$	$0 \rightsquigarrow^{\wedge} 4$
5)	$\boxed{a_0{:}\, \Diamond_R \top}$	$3 \rightsquigarrow^{\wedge} 5$
6)	$a_0{:}\, \mathsf{A}\Box_{R-} p$	$3 \rightsquigarrow^{\wedge} 6$
7)	$a_0{:}\, \Diamond_R a_1$	$5 \rightsquigarrow^{\Diamond} 7$
8)	$a_1{:}\, \top$	$5 \rightsquigarrow^{\Diamond} 8$
9)	$a_1{:}\, \Box_{R-} p$	$(6, 7) \rightsquigarrow^{\mathsf{A}} 9$

0)	$a_0 : F$	
1)	$\mathrm{Trans}(R)$	
2)	$R \sqsubseteq R$	
3)	$a_0 : (\Diamond_R \top \wedge \mathsf{A}\Box_{R-} p)$	$0 \leadsto^{\wedge} 3$
4)	$a_0 : \Box_R G$	$0 \leadsto^{\wedge} 4$
5)	$\boxed{a_0 : \Diamond_R \top}$	$3 \leadsto^{\wedge} 5$
6)	$a_0 : \mathsf{A}\Box_{R-} p$	$3 \leadsto^{\wedge} 6$
7)	$a_0 : \Diamond_R a_1$	$5 \leadsto^{\Diamond} 7$
8)	$a_1 : \top$	$5 \leadsto^{\Diamond} 8$
9)	$a_1 : \Box_{R-} p$	$(6,7) \leadsto^{\mathsf{A}} 9$
10)	$a_0 : \Box_{R-} p$	$(6,0) \leadsto^{\mathsf{A}} 10$

$F = \Diamond_R \top \wedge \mathsf{A}\Box_{R-}\, p \wedge \Box_R G$, where $G = \downarrow x.\Diamond_R \downarrow y.x\colon \Diamond_R \neg y$ and R is transitive

0)	$a_0\colon F$	
1)	$\mathrm{Trans}(R)$	
2)	$R \sqsubseteq R$	
3)	$a_0\colon (\Diamond_R \top \wedge \mathsf{A}\Box_{R-}\, p)$	$0 \rightsquigarrow^{\wedge} 3$
4)	$a_0\colon \Box_R G$	$0 \rightsquigarrow^{\wedge} 4$
5)	$\boxed{a_0\colon \Diamond_R \top}$	$3 \rightsquigarrow^{\wedge} 5$
6)	$a_0\colon \mathsf{A}\Box_{R-}\, p$	$3 \rightsquigarrow^{\wedge} 6$
7)	$a_0\colon \Diamond_R a_1$	$5 \rightsquigarrow^{\Diamond} 7$
8)	$a_1\colon \top$	$5 \rightsquigarrow^{\Diamond} 8$
9)	$a_1\colon \Box_{R-}\, p$	$(6,7) \rightsquigarrow^{\mathsf{A}} 9$
10)	$a_0\colon \Box_{R-}\, p$	$(6,0) \rightsquigarrow^{\mathsf{A}} 10$
11)	$a_1\colon \Box_R G$	$(4,7,1,2) \rightsquigarrow^{\mathrm{Trans}} 11$

$F = \Diamond_R \top \wedge \mathsf{A}\Box_{R^-} p \wedge \Box_R G$, where $G = \downarrow x.\Diamond_R \downarrow y.x\colon \Diamond_R \neg y$ and R is transitive

0)	$a_0 : F$	
1)	$\mathrm{Trans}(R)$	
2)	$R \sqsubseteq R$	
3)	$a_0 : (\Diamond_R \top \wedge \mathsf{A}\Box_{R^-} p)$	$0 \leadsto^{\wedge} 3$
4)	$a_0 : \Box_R G$	$0 \leadsto^{\wedge} 4$
5)	$\boxed{a_0 : \Diamond_R \top}$	$3 \leadsto^{\wedge} 5$
6)	$a_0 : \mathsf{A}\Box_{R^-} p$	$3 \leadsto^{\wedge} 6$
7)	$a_0 : \Diamond_R a_1$	$5 \leadsto^{\Diamond} 7$
8)	$a_1 : \top$	$5 \leadsto^{\Diamond} 8$
9)	$a_1 : \Box_{R^-} p$	$(6,7) \leadsto^{\mathsf{A}} 9$
10)	$a_0 : \Box_{R^-} p$	$(6,0) \leadsto^{\mathsf{A}} 10$
11)	$a_1 : \Box_R G$	$(4,7,1,2) \leadsto^{\mathrm{Trans}} 11$
12)	$a_1 : G$	$(4,7) \leadsto^{\Box} 12$

$F = \Diamond_R \top \wedge \mathsf{A}\Box_{R-}\, p \wedge \Box_R G$, where $G = \downarrow x.\Diamond_R \downarrow y.x{:}\, \Diamond_R \neg\, y$ and R is transitive

0)	$a_0 : F$	
1)	$\mathrm{Trans}(R)$	
2)	$R \sqsubseteq R$	
3)	$a_0 : (\Diamond_R \top \wedge \mathsf{A}\Box_{R-}\, p)$	$0 \leadsto^{\wedge} 3$
4)	$a_0 : \Box_R G$	$0 \leadsto^{\wedge} 4$
5)	$\boxed{a_0 : \Diamond_R \top}$	$3 \leadsto^{\wedge} 5$
6)	$a_0 : \mathsf{A}\Box_{R-}\, p$	$3 \leadsto^{\wedge} 6$
7)	$a_0 : \Diamond_R a_1$	$5 \leadsto^{\Diamond} 7$
8)	$a_1 : \top$	$5 \leadsto^{\Diamond} 8$
9)	$a_1 : \Box_{R-}\, p$	$(6,7) \leadsto^{\mathsf{A}} 9$
10)	$a_0 : \Box_{R-}\, p$	$(6,0) \leadsto^{\mathsf{A}} 10$
11)	$a_1 : \Box_R G$	$(4,7,1,2) \leadsto^{\mathrm{Trans}} 11$
12)	$a_1 : G$	$(4,7) \leadsto^{\Box} 12$
13)	$a_0 : p$	$(9,7) \leadsto^{\Box} 13$

$F = \Diamond_R\top \wedge \mathsf{A}\Box_{R^-}p \wedge \Box_R G$, where $G = \downarrow x.\Diamond_R\downarrow y.x{:}\,\Diamond_R\neg y$ and R is transitive

0)	$a_0 : F$	
1)	$\mathrm{Trans}(R)$	
2)	$R \sqsubseteq R$	
3)	$a_0 : (\Diamond_R\top \wedge \mathsf{A}\Box_{R^-}p)$	$0 \leadsto^{\wedge} 3$
4)	$a_0 : \Box_R G$	$0 \leadsto^{\wedge} 4$
5)	$\boxed{a_0 : \Diamond_R\top}$	$3 \leadsto^{\wedge} 5$
6)	$a_0 : \mathsf{A}\Box_{R^-}p$	$3 \leadsto^{\wedge} 6$
7)	$a_0 : \Diamond_R a_1$	$5 \leadsto^{\Diamond} 7$
8)	$a_1 : \top$	$5 \leadsto^{\Diamond} 8$
9)	$a_1 : \Box_{R^-}p$	$(6,7) \leadsto^{\mathsf{A}} 9$
10)	$a_0 : \Box_{R^-}p$	$(6,0) \leadsto^{\mathsf{A}} 10$
11)	$a_1 : \Box_R G$	$(4,7,1,2) \leadsto^{\mathrm{Trans}} 11$
12)	$a_1 : G$	$(4,7) \leadsto^{\Box} 12$
13)	$a_0 : p$	$(9,7) \leadsto^{\Box} 13$
14)	$\boxed{a_1 : \Diamond_R\downarrow y.a_1 : \Diamond_R\neg y}$	$12 \leadsto^{\downarrow} 14$

$F = \Diamond_R \top \wedge \mathsf{A}\Box_{R-}\, p \wedge \Box_R G$, where $G = \downarrow x.\Diamond_R \downarrow y.x\colon \Diamond_R \neg\, y$ and R is transitive

0)	$a_0\colon F$	
1)	$\mathrm{Trans}(R)$	
2)	$R \sqsubseteq R$	
3)	$a_0\colon (\Diamond_R \top \wedge \mathsf{A}\Box_{R-}\, p)$	$0 \leadsto^{\wedge} 3$
4)	$a_0\colon \Box_R G$	$0 \leadsto^{\wedge} 4$
5)	$\boxed{a_0\colon \Diamond_R \top}$	$3 \leadsto^{\wedge} 5$
6)	$a_0\colon \mathsf{A}\Box_{R-}\, p$	$3 \leadsto^{\wedge} 6$
7)	$a_0\colon \Diamond_R a_1$	$5 \leadsto^{\Diamond} 7$
8)	$a_1\colon \top$	$5 \leadsto^{\Diamond} 8$
9)	$a_1\colon \Box_{R-}\, p$	$(6,7) \leadsto^{\mathsf{A}} 9$
10)	$a_0\colon \Box_{R-}\, p$	$(6,0) \leadsto^{\mathsf{A}} 10$
11)	$a_1\colon \Box_R G$	$(4,7,1,2) \leadsto^{\mathrm{Trans}} 11$
12)	$a_1\colon G$	$(4,7) \leadsto^{\Box} 12$
13)	$a_0\colon p$	$(9,7) \leadsto^{\Box} 13$
14)	$\boxed{a_1\colon \Diamond_R \downarrow y.a_1\colon \Diamond_R \neg\, y}$	$12 \leadsto^{\downarrow} 14$
15)	$a_1\colon \Diamond_R a_2$	$14 \leadsto^{\Diamond} 15$
16)	$a_2\colon \downarrow y.a_1\colon \Diamond_R \neg\, y$	$14 \leadsto^{\Diamond} 16$

$F = \Diamond_R \top \wedge \mathsf{A}\Box_{R^-} p \wedge \Box_R G$, where $G = {\downarrow}x.\Diamond_R{\downarrow}y.x{:}\,\Diamond_R \neg y$ and R is transitive

0)	$a_0 : F$	
1)	$\mathrm{Trans}(R)$	
2)	$R \sqsubseteq R$	
3)	$a_0 : (\Diamond_R \top \wedge \mathsf{A}\Box_{R^-} p)$	$0 \rightsquigarrow^{\wedge} 3$
4)	$a_0 : \Box_R G$	$0 \rightsquigarrow^{\wedge} 4$
5)	$\boxed{a_0 : \Diamond_R \top}$	$3 \rightsquigarrow^{\wedge} 5$
6)	$a_0 : \mathsf{A}\Box_{R^-} p$	$3 \rightsquigarrow^{\wedge} 6$
7)	$a_0 : \Diamond_R a_1$	$5 \rightsquigarrow^{\Diamond} 7$
8)	$a_1 : \top$	$5 \rightsquigarrow^{\Diamond} 8$
9)	$a_1 : \Box_{R^-} p$	$(6,7) \rightsquigarrow^{\mathsf{A}} 9$
10)	$a_0 : \Box_{R^-} p$	$(6,0) \rightsquigarrow^{\mathsf{A}} 10$
11)	$a_1 : \Box_R G$	$(4,7,1,2) \rightsquigarrow^{\mathrm{Trans}} 11$
12)	$a_1 : G$	$(4,7) \rightsquigarrow^{\Box} 12$
13)	$a_0 : p$	$(9,7) \rightsquigarrow^{\Box} 13$
14)	$\boxed{a_1 : \Diamond_R {\downarrow} y.a_1 : \Diamond_R \neg y}$	$12 \rightsquigarrow^{\downarrow} 14$
15)	$a_1 : \Diamond_R a_2$	$14 \rightsquigarrow^{\Diamond} 15$
16)	$a_2 : {\downarrow} y.a_1 : \Diamond_R \neg y$	$14 \rightsquigarrow^{\Diamond} 16$
17)	$a_2 : \Box_R G$	$(11,15,1,2) \rightsquigarrow^{\mathrm{Trans}} 17$

$F = \Diamond_R \top \wedge \mathsf{A}\Box_{R^-} p \wedge \Box_R G$, where $G = \downarrow x.\Diamond_R \downarrow y.x{:}\,\Diamond_R \neg y$ and R is transitive

0)	$a_0 : F$	
1)	$\mathrm{Trans}(R)$	
2)	$R \sqsubseteq R$	
3)	$a_0 : (\Diamond_R \top \wedge \mathsf{A}\Box_{R^-} p)$	$0 \leadsto^{\wedge} 3$
4)	$a_0 : \Box_R G$	$0 \leadsto^{\wedge} 4$
5)	$\boxed{a_0 : \Diamond_R \top}$	$3 \leadsto^{\wedge} 5$
6)	$a_0 : \mathsf{A}\Box_{R^-} p$	$3 \leadsto^{\wedge} 6$
7)	$a_0 : \Diamond_R a_1$	$5 \leadsto^{\Diamond} 7$
8)	$a_1 : \top$	$5 \leadsto^{\Diamond} 8$
9)	$a_1 : \Box_{R^-} p$	$(6,7) \leadsto^{\mathsf{A}} 9$
10)	$a_0 : \Box_{R^-} p$	$(6,0) \leadsto^{\mathsf{A}} 10$
11)	$a_1 : \Box_R G$	$(4,7,1,2) \leadsto^{\mathrm{Trans}} 11$
12)	$a_1 : G$	$(4,7) \leadsto^{\Box} 12$
13)	$a_0 : p$	$(9,7) \leadsto^{\Box} 13$
14)	$\boxed{a_1 : \Diamond_R \downarrow y.a_1 : \Diamond_R \neg y}$	$12 \leadsto^{\downarrow} 14$
15)	$a_1 : \Diamond_R a_2$	$14 \leadsto^{\Diamond} 15$
16)	$a_2 : \downarrow y.a_1 : \Diamond_R \neg y$	$14 \leadsto^{\Diamond} 16$
17)	$a_2 : \Box_R G$	$(11,15,1,2) \leadsto^{\mathrm{Trans}} 17$
18)	$a_2 : G$	$(11,15) \leadsto^{\Box} 18$

$F = \Diamond_R \top \wedge \mathsf{A}\Box_{R^-} p \wedge \Box_R G$, where $G = \downarrow x.\Diamond_R \downarrow y.x{:}\,\Diamond_R \neg y$ and R is transitive

0)	$a_0 : F$	
1)	$\mathrm{Trans}(R)$	
2)	$R \sqsubseteq R$	
3)	$a_0 : (\Diamond_R \top \wedge \mathsf{A}\Box_{R^-} p)$	$0 \leadsto^{\wedge} 3$
4)	$a_0 : \Box_R G$	$0 \leadsto^{\wedge} 4$
5)	$\boxed{a_0 : \Diamond_R \top}$	$3 \leadsto^{\wedge} 5$
6)	$a_0 : \mathsf{A}\Box_{R^-} p$	$3 \leadsto^{\wedge} 6$
7)	$a_0 : \Diamond_R a_1$	$5 \leadsto^{\Diamond} 7$
8)	$a_1 : \top$	$5 \leadsto^{\Diamond} 8$
9)	$a_1 : \Box_{R^-} p$	$(6,7) \leadsto^{\mathsf{A}} 9$
10)	$a_0 : \Box_{R^-} p$	$(6,0) \leadsto^{\mathsf{A}} 10$
11)	$a_1 : \Box_R G$	$(4,7,1,2) \leadsto^{\mathrm{Trans}} 11$
12)	$a_1 : G$	$(4,7) \leadsto^{\Box} 12$
13)	$a_0 : p$	$(9,7) \leadsto^{\Box} 13$
14)	$\boxed{a_1 : \Diamond_R \downarrow y.a_1 : \Diamond_R \neg y}$	$12 \leadsto^{\downarrow} 14$
15)	$a_1 : \Diamond_R a_2$	$14 \leadsto^{\Diamond} 15$
16)	$a_2 : \downarrow y.a_1 : \Diamond_R \neg y$	$14 \leadsto^{\Diamond} 16$
17)	$a_2 : \Box_R G$	$(11,15,1,2) \leadsto^{\mathrm{Trans}} 17$
18)	$a_2 : G$	$(11,15) \leadsto^{\Box} 18$
19)	$a_2 : a_1 : \Diamond_R \neg a_2$	$16 \leadsto^{\downarrow} 19$

$F = \Diamond_R \top \wedge \mathsf{A}\Box_{R-}\, p \wedge \Box_R G$, where $G = \downarrow x.\Diamond_R \downarrow y.x\colon \Diamond_R \neg y$ and R is transitive

0)	$a_0\colon F$	
1)	$\mathrm{Trans}(R)$	
2)	$R \sqsubseteq R$	
3)	$a_0\colon (\Diamond_R \top \wedge \mathsf{A}\Box_{R-}\, p)$	$0 \rightsquigarrow^{\wedge} 3$
4)	$a_0\colon \Box_R G$	$0 \rightsquigarrow^{\wedge} 4$
5)	$a_0\colon \Diamond_R \top$	$3 \rightsquigarrow^{\wedge} 5$
6)	$a_0\colon \mathsf{A}\Box_{R-}\, p$	$3 \rightsquigarrow^{\wedge} 6$
7)	$a_0\colon \Diamond_R a_1$	$5 \rightsquigarrow^{\Diamond} 7$
8)	$a_1\colon \top$	$5 \rightsquigarrow^{\Diamond} 8$
9)	$a_1\colon \Box_{R-}\, p$	$(6,7) \rightsquigarrow^{\mathsf{A}} 9$
10)	$a_0\colon \Box_{R-}\, p$	$(6,0) \rightsquigarrow^{\mathsf{A}} 10$
11)	$a_1\colon \Box_R G$	$(4,7,1,2) \rightsquigarrow^{\mathrm{Trans}} 11$
12)	$a_1\colon G$	$(4,7) \rightsquigarrow^{\Box} 12$
13)	$a_0\colon p$	$(9,7) \rightsquigarrow^{\Box} 13$
14)	$a_1\colon \Diamond_R \downarrow y.a_1\colon \Diamond_R \neg y$	$12 \rightsquigarrow^{\downarrow} 14$
15)	$a_1\colon \Diamond_R a_2$	$14 \rightsquigarrow^{\Diamond} 15$
16)	$a_2\colon \downarrow y.a_1\colon \Diamond_R \neg y$	$14 \rightsquigarrow^{\Diamond} 16$
17)	$a_2\colon \Box_R G$	$(11,15,1,2) \rightsquigarrow^{\mathrm{Trans}} 17$
18)	$a_2\colon G$	$(11,15) \rightsquigarrow^{\Box} 18$
19)	$a_2\colon a_1\colon \Diamond_R \neg a_2$	$16 \rightsquigarrow^{\downarrow} 19$
20)	$a_2\colon \Diamond_R \downarrow y.a_2\colon \Diamond_R \neg y$	$18 \rightsquigarrow^{\downarrow} 20$

$F = \Diamond_R \top \wedge \mathsf{A}\Box_{R^-} p \wedge \Box_R G$, where $G = \downarrow x.\Diamond_R \downarrow y.x\colon \Diamond_R \neg y$ and R is transitive

0)	$a_0 : F$	
1)	$\mathrm{Trans}(R)$	
2)	$R \sqsubseteq R$	
3)	$a_0 : (\Diamond_R \top \wedge \mathsf{A}\Box_{R^-} p)$	$0 \leadsto^{\wedge} 3$
4)	$a_0 : \Box_R G$	$0 \leadsto^{\wedge} 4$
5)	$\boxed{a_0 : \Diamond_R \top}$	$3 \leadsto^{\wedge} 5$
6)	$a_0 : \mathsf{A}\Box_{R^-} p$	$3 \leadsto^{\wedge} 6$
7)	$a_0 : \Diamond_R a_1$	$5 \leadsto^{\Diamond} 7$
8)	$a_1 : \top$	$5 \leadsto^{\Diamond} 8$
9)	$a_1 : \Box_{R^-} p$	$(6,7) \leadsto^{\mathsf{A}} 9$
10)	$a_0 : \Box_{R^-} p$	$(6,0) \leadsto^{\mathsf{A}} 10$
11)	$a_1 : \Box_R G$	$(4,7,1,2) \leadsto^{\mathrm{Trans}} 11$
12)	$a_1 : G$	$(4,7) \leadsto^{\Box} 12$
13)	$a_0 : p$	$(9,7) \leadsto^{\Box} 13$
14)	$\boxed{a_1 : \Diamond_R \downarrow y.a_1 : \Diamond_R \neg y}$	$12 \leadsto^{\downarrow} 14$
15)	$a_1 : \Diamond_R a_2$	$14 \leadsto^{\Diamond} 15$
16)	$a_2 : \downarrow y.a_1 : \Diamond_R \neg y$	$14 \leadsto^{\Diamond} 16$
17)	$a_2 : \Box_R G$	$(11,15,1,2) \leadsto^{\mathrm{Trans}} 17$
18)	$a_2 : G$	$(11,15) \leadsto^{\Box} 18$
19)	$a_2 : a_1 : \Diamond_R \neg a_2$	$16 \leadsto^{\downarrow} 19$
20)	$\boxed{a_2 : \Diamond_R \downarrow y.a_2 : \Diamond_R \neg y}$	$18 \leadsto^{\downarrow} 20$
21)	$\boxed{a_1 : \Diamond_R \neg a_2}$	$19 \leadsto^{@} 21$

$F = \Diamond_R \top \wedge \mathsf{A}\Box_{R^-} p \wedge \Box_R G$, where $G = {\downarrow}x.\Diamond_R{\downarrow}y.x\colon \Diamond_R \neg y$ and R is transitive

0)	$a_0\colon F$	
1)	$\mathrm{Trans}(R)$	
2)	$R \sqsubseteq R$	
3)	$a_0\colon (\Diamond_R \top \wedge \mathsf{A}\Box_{R^-} p)$	$0 \rightsquigarrow^{\wedge} 3$
4)	$a_0\colon \Box_R G$	$0 \rightsquigarrow^{\wedge} 4$
5)	$\boxed{a_0\colon \Diamond_R \top}$	$3 \rightsquigarrow^{\wedge} 5$
6)	$a_0\colon \mathsf{A}\Box_{R^-} p$	$3 \rightsquigarrow^{\wedge} 6$
7)	$a_0\colon \Diamond_R a_1$	$5 \rightsquigarrow^{\Diamond} 7$
8)	$a_1\colon \top$	$5 \rightsquigarrow^{\Diamond} 8$
9)	$a_1\colon \Box_{R^-} p$	$(6,7) \rightsquigarrow^{\mathsf{A}} 9$
10)	$a_0\colon \Box_{R^-} p$	$(6,0) \rightsquigarrow^{\mathsf{A}} 10$
11)	$a_1\colon \Box_R G$	$(4,7,1,2) \rightsquigarrow^{\mathrm{Trans}} 11$
12)	$a_1\colon G$	$(4,7) \rightsquigarrow^{\Box} 12$
13)	$a_0\colon p$	$(9,7) \rightsquigarrow^{\Box} 13$
14)	$\boxed{a_1\colon \Diamond_R{\downarrow}y.a_1\colon \Diamond_R \neg y}$	$12 \rightsquigarrow^{\downarrow} 14$
15)	$a_1\colon \Diamond_R a_2$	$14 \rightsquigarrow^{\Diamond} 15$
16)	$a_2\colon {\downarrow}y.a_1\colon \Diamond_R \neg y$	$14 \rightsquigarrow^{\Diamond} 16$
17)	$a_2\colon \Box_R G$	$(11,15,1,2) \rightsquigarrow^{\mathrm{Trans}} 17$
18)	$a_2\colon G$	$(11,15) \rightsquigarrow^{\Box} 18$
19)	$a_2\colon a_1\colon \Diamond_R \neg a_2$	$16 \rightsquigarrow^{\downarrow} 19$
20)	$\boxed{a_2\colon \Diamond_R{\downarrow}y.a_2\colon \Diamond_R \neg y}$	$18 \rightsquigarrow^{\downarrow} 20$
21)	$\boxed{a_1\colon \Diamond_R \neg a_2}$	$19 \rightsquigarrow^{@} 21$
22)	$a_2\colon \Diamond_R a_3$	$20 \rightsquigarrow^{\Diamond} 22$
23)	$a_3\colon {\downarrow}y.a_2\colon \Diamond_R \neg y$	$20 \rightsquigarrow^{\Diamond} 23$

$F = \Diamond_R\top \wedge \mathsf{A}\Box_{R-}p \wedge \Box_R G$, where $G = \downarrow x.\Diamond_R{\downarrow}y.x{:}\,\Diamond_R\neg y$ and R is transitive

0)	$a_0 : F$	
1)	$\mathrm{Trans}(R)$	
2)	$R \sqsubseteq R$	
3)	$a_0 : (\Diamond_R\top \wedge \mathsf{A}\Box_{R-}p)$	$0 \leadsto^{\wedge} 3$
4)	$a_0 : \Box_R G$	$0 \leadsto^{\wedge} 4$
5)	$\boxed{a_0 : \Diamond_R\top}$	$3 \leadsto^{\wedge} 5$
6)	$a_0 : \mathsf{A}\Box_{R-}p$	$3 \leadsto^{\wedge} 6$
7)	$a_0 : \Diamond_R a_1$	$5 \leadsto^{\Diamond} 7$
8)	$a_1 : \top$	$5 \leadsto^{\Diamond} 8$
9)	$a_1 : \Box_{R-}p$	$(6,7) \leadsto^{\mathsf{A}} 9$
10)	$a_0 : \Box_{R-}p$	$(6,0) \leadsto^{\mathsf{A}} 10$
11)	$a_1 : \Box_R G$	$(4,7,1,2) \leadsto^{\mathrm{Trans}} 11$
12)	$a_1 : G$	$(4,7) \leadsto^{\Box} 12$
13)	$a_0 : p$	$(9,7) \leadsto^{\Box} 13$
14)	$\boxed{a_1 : \Diamond_R{\downarrow}y.a_1 : \Diamond_R\neg y}$	$12 \leadsto^{\downarrow} 14$
15)	$a_1 : \Diamond_R a_2$	$14 \leadsto^{\Diamond} 15$
16)	$a_2 : {\downarrow}y.a_1 : \Diamond_R\neg y$	$14 \leadsto^{\Diamond} 16$
17)	$a_2 : \Box_R G$	$(11,15,1,2) \leadsto^{\mathrm{Trans}} 17$
18)	$a_2 : G$	$(11,15) \leadsto^{\Box} 18$
19)	$a_2 : a_1 : \Diamond_R\neg a_2$	$16 \leadsto^{\downarrow} 19$
20)	$\boxed{a_2 : \Diamond_R{\downarrow}y.a_2 : \Diamond_R\neg y}$	$18 \leadsto^{\downarrow} 20$
21)	$\boxed{a_1 : \Diamond_R\neg a_2}$	$19 \leadsto^{@} 21$
22)	$a_2 : \Diamond_R a_3$	$20 \leadsto^{\Diamond} 22$
23)	$a_3 : {\downarrow}y.a_2 : \Diamond_R\neg y$	$20 \leadsto^{\Diamond} 23$
24)	$a_1 : \Diamond_R a_4$	$21 \leadsto^{\Diamond} 24$
25)	$a_4 : \neg a_2$	$21 \leadsto^{\Diamond} 25$

$F = \Diamond_R \top \wedge \mathsf{A}\Box_{R^-} p \wedge \Box_R G$, where $G = \downarrow x.\Diamond_R \downarrow y.x\colon \Diamond_R \neg y$ and R is transitive

0)	$a_0 : F$	
1)	$\mathrm{Trans}(R)$	
2)	$R \sqsubseteq R$	
3)	$a_0 : (\Diamond_R \top \wedge \mathsf{A}\Box_{R^-} p)$	$0 \leadsto^{\wedge} 3$
4)	$a_0 : \Box_R G$	$0 \leadsto^{\wedge} 4$
5)	$\boxed{a_0 : \Diamond_R \top}$	$3 \leadsto^{\wedge} 5$
6)	$a_0 : \mathsf{A}\Box_{R^-} p$	$3 \leadsto^{\wedge} 6$
7)	$a_0 : \Diamond_R a_1$	$5 \leadsto^{\Diamond} 7$
8)	$a_1 : \top$	$5 \leadsto^{\Diamond} 8$
9)	$a_1 : \Box_{R^-} p$	$(6,7) \leadsto^{\mathsf{A}} 9$
10)	$a_0 : \Box_{R^-} p$	$(6,0) \leadsto^{\mathsf{A}} 10$
11)	$a_1 : \Box_R G$	$(4,7,1,2) \leadsto^{\mathrm{Trans}} 11$
12)	$a_1 : G$	$(4,7) \leadsto^{\Box} 12$
13)	$a_0 : p$	$(9,7) \leadsto^{\Box} 13$
14)	$\boxed{a_1 : \Diamond_R \downarrow y.a_1 : \Diamond_R \neg y}$	$12 \leadsto^{\downarrow} 14$
15)	$a_1 : \Diamond_R a_2$	$14 \leadsto^{\Diamond} 15$
16)	$a_2 : \downarrow y.a_1 : \Diamond_R \neg y$	$14 \leadsto^{\Diamond} 16$
17)	$a_2 : \Box_R G$	$(11,15,1,2) \leadsto^{\mathrm{Trans}} 17$
18)	$a_2 : G$	$(11,15) \leadsto^{\Box} 18$
19)	$a_2 : a_1 : \Diamond_R \neg a_2$	$16 \leadsto^{\downarrow} 19$
20)	$\boxed{a_2 : \Diamond_R \downarrow y.a_2 : \Diamond_R \neg y}$	$18 \leadsto^{\downarrow} 20$
21)	$\boxed{a_1 : \Diamond_R \neg a_2}$	$19 \leadsto^{@} 21$
22)	$a_2 : \Diamond_R a_3$	$20 \leadsto^{\Diamond} 22$
23)	$a_3 : \downarrow y.a_2 : \Diamond_R \neg y$	$20 \leadsto^{\Diamond} 23$
24)	$a_1 : \Diamond_R a_4$	$21 \leadsto^{\Diamond} 24$
25)	$a_4 : \neg a_2$	$21 \leadsto^{\Diamond} 25$
26)	$a_3 : \Box_R G$	$(17,22,1,2) \leadsto^{\mathrm{Trans}} 26$

$F = \Diamond_R \top \wedge \mathsf{A}\Box_{R^-} p \wedge \Box_R G$, where $G = \downarrow x.\Diamond_R \downarrow y.x{:}\Diamond_R \neg y$ and R is transitive

0)	$a_0 : F$	
1)	Trans(R)	
2)	$R \sqsubseteq R$	
3)	$a_0 : (\Diamond_R \top \wedge \mathsf{A}\Box_{R^-} p)$	$0 \leadsto^{\wedge} 3$
4)	$a_0 : \Box_R G$	$0 \leadsto^{\wedge} 4$
5)	$a_0 : \Diamond_R \top$	$3 \leadsto^{\wedge} 5$
6)	$a_0 : \mathsf{A}\Box_{R^-} p$	$3 \leadsto^{\wedge} 6$
7)	$a_0 : \Diamond_R a_1$	$5 \leadsto^{\Diamond} 7$
8)	$a_1 : \top$	$5 \leadsto^{\Diamond} 8$
9)	$a_1 : \Box_{R^-} p$	$(6,7) \leadsto^{\mathsf{A}} 9$
10)	$a_0 : \Box_{R^-} p$	$(6,0) \leadsto^{\mathsf{A}} 10$
11)	$a_1 : \Box_R G$	$(4,7,1,2) \leadsto^{\mathrm{Trans}} 11$
12)	$a_1 : G$	$(4,7) \leadsto^{\Box} 12$
13)	$a_0 : p$	$(9,7) \leadsto^{\Box} 13$
14)	$a_1 : \Diamond_R \downarrow y.a_1 : \Diamond_R \neg y$	$12 \leadsto^{\downarrow} 14$
15)	$a_1 : \Diamond_R a_2$	$14 \leadsto^{\Diamond} 15$
16)	$a_2 : \downarrow y.a_1 : \Diamond_R \neg y$	$14 \leadsto^{\Diamond} 16$
17)	$a_2 : \Box_R G$	$(11,15,1,2) \leadsto^{\mathrm{Trans}} 17$
18)	$a_2 : G$	$(11,15) \leadsto^{\Box} 18$
19)	$a_2 : a_1 : \Diamond_R \neg a_2$	$16 \leadsto^{\downarrow} 19$
20)	$a_2 : \Diamond_R \downarrow y.a_2 : \Diamond_R \neg y$	$18 \leadsto^{\downarrow} 20$
21)	$a_1 : \Diamond_R \neg a_2$	$19 \leadsto^{@} 21$
22)	$a_2 : \Diamond_R a_3$	$20 \leadsto^{\Diamond} 22$
23)	$a_3 : \downarrow y.a_2 : \Diamond_R \neg y$	$20 \leadsto^{\Diamond} 23$
24)	$a_1 : \Diamond_R a_4$	$21 \leadsto^{\Diamond} 24$
25)	$a_4 : \neg a_2$	$21 \leadsto^{\Diamond} 25$
26)	$a_3 : \Box_R G$	$(17,22,1,2) \leadsto^{\mathrm{Trans}} 26$
27)	$a_3 : G$	$(17,22) \leadsto^{\Box} 27$

$F = \Diamond_R \top \wedge \mathrm{A}\Box_{R^-} p \wedge \Box_R G$, where $G = \downarrow x.\Diamond_R \downarrow y.x{:}\,\Diamond_R \neg y$ and R is transitive

0)	$a_0 : F$	
1)	$\mathrm{Trans}(R)$	
2)	$R \sqsubseteq R$	
3)	$a_0 : (\Diamond_R \top \wedge \mathrm{A}\Box_{R^-} p)$	$0 \rightsquigarrow^{\wedge} 3$
4)	$a_0 : \Box_R G$	$0 \rightsquigarrow^{\wedge} 4$
5)	$\boxed{a_0 : \Diamond_R \top}$	$3 \rightsquigarrow^{\wedge} 5$
6)	$a_0 : \mathrm{A}\Box_{R^-} p$	$3 \rightsquigarrow^{\wedge} 6$
7)	$a_0 : \Diamond_R a_1$	$5 \rightsquigarrow^{\Diamond} 7$
8)	$a_1 : \top$	$5 \rightsquigarrow^{\Diamond} 8$
9)	$a_1 : \Box_{R^-} p$	$(6,7) \rightsquigarrow^{\mathrm{A}} 9$
10)	$a_0 : \Box_{R^-} p$	$(6,0) \rightsquigarrow^{\mathrm{A}} 10$
11)	$a_1 : \Box_R G$	$(4,7,1,2) \rightsquigarrow^{\mathrm{Trans}} 11$
12)	$a_1 : G$	$(4,7) \rightsquigarrow^{\Box} 12$
13)	$a_0 : p$	$(9,7) \rightsquigarrow^{\Box} 13$
14)	$\boxed{a_1 : \Diamond_R \downarrow y.a_1 : \Diamond_R \neg y}$	$12 \rightsquigarrow^{\downarrow} 14$
15)	$a_1 : \Diamond_R a_2$	$14 \rightsquigarrow^{\Diamond} 15$
16)	$a_2 : \downarrow y.a_1 : \Diamond_R \neg y$	$14 \rightsquigarrow^{\Diamond} 16$
17)	$a_2 : \Box_R G$	$(11,15,1,2) \rightsquigarrow^{\mathrm{Trans}} 17$
18)	$a_2 : G$	$(11,15) \rightsquigarrow^{\Box} 18$
19)	$a_2 : a_1 : \Diamond_R \neg a_2$	$16 \rightsquigarrow^{\downarrow} 19$
20)	$\boxed{a_2 : \Diamond_R \downarrow y.a_2 : \Diamond_R \neg y}$	$18 \rightsquigarrow^{\downarrow} 20$
21)	$\boxed{a_1 : \Diamond_R \neg a_2}$	$19 \rightsquigarrow^{@} 21$
22)	$a_2 : \Diamond_R a_3$	$20 \rightsquigarrow^{\Diamond} 22$
23)	$a_3 : \downarrow y.a_2 : \Diamond_R \neg y$	$20 \rightsquigarrow^{\Diamond} 23$
24)	$a_1 : \Diamond_R a_4$	$21 \rightsquigarrow^{\Diamond} 24$
25)	$a_4 : \neg a_2$	$21 \rightsquigarrow^{\Diamond} 25$
26)	$a_3 : \Box_R G$	$(17,22,1,2) \rightsquigarrow^{\mathrm{Trans}} 26$
27)	$a_3 : G$	$(17,22) \rightsquigarrow^{\Box} 27$
28)	$a_3 : a_2 : \Diamond_R \neg a_3$	$23 \rightsquigarrow^{\downarrow} 28$

$F = \Diamond_R \top \wedge \mathsf{A}\Box_{R^-} p \wedge \Box_R G$, where $G = \downarrow x.\Diamond_R \downarrow y.x{:}\,\Diamond_R \neg y$ and R is transitive

0)	$a_0 : F$	
1)	$\mathrm{Trans}(R)$	
2)	$R \sqsubseteq R$	
3)	$a_0 : (\Diamond_R \top \wedge \mathsf{A}\Box_{R^-} p)$	$0 \leadsto^{\wedge} 3$
4)	$a_0 : \Box_R G$	$0 \leadsto^{\wedge} 4$
5)	$a_0 : \Diamond_R \top$	$3 \leadsto^{\wedge} 5$
6)	$a_0 : \mathsf{A}\Box_{R^-} p$	$3 \leadsto^{\wedge} 6$
7)	$a_0 : \Diamond_R a_1$	$5 \leadsto^{\Diamond} 7$
8)	$a_1 : \top$	$5 \leadsto^{\Diamond} 8$
9)	$a_1 : \Box_{R^-} p$	$(6,7) \leadsto^{\mathsf{A}} 9$
10)	$a_0 : \Box_{R^-} p$	$(6,0) \leadsto^{\mathsf{A}} 10$
11)	$a_1 : \Box_R G$	$(4,7,1,2) \leadsto^{\mathrm{Trans}} 11$
12)	$a_1 : G$	$(4,7) \leadsto^{\Box} 12$
13)	$a_0 : p$	$(9,7) \leadsto^{\Box} 13$
14)	$a_1 : \Diamond_R \downarrow y.a_1 : \Diamond_R \neg y$	$12 \leadsto^{\downarrow} 14$
15)	$a_1 : \Diamond_R a_2$	$14 \leadsto^{\Diamond} 15$
16)	$a_2 : \downarrow y.a_1 : \Diamond_R \neg y$	$14 \leadsto^{\Diamond} 16$
17)	$a_2 : \Box_R G$	$(11,15,1,2) \leadsto^{\mathrm{Trans}} 17$
18)	$a_2 : G$	$(11,15) \leadsto^{\Box} 18$
19)	$a_2 : a_1 : \Diamond_R \neg a_2$	$16 \leadsto^{\downarrow} 19$
20)	$a_2 : \Diamond_R \downarrow y.a_2 : \Diamond_R \neg y$	$18 \leadsto^{\downarrow} 20$
21)	$a_1 : \Diamond_R \neg a_2$	$19 \leadsto^{@} 21$
22)	$a_2 : \Diamond_R a_3$	$20 \leadsto^{\Diamond} 22$
23)	$a_3 : \downarrow y.a_2 : \Diamond_R \neg y$	$20 \leadsto^{\Diamond} 23$
24)	$a_1 : \Diamond_R a_4$	$21 \leadsto^{\Diamond} 24$
25)	$a_4 : \neg a_2$	$21 \leadsto^{\Diamond} 25$
26)	$a_3 : \Box_R G$	$(17,22,1,2) \leadsto^{\mathrm{Trans}} 26$
27)	$a_3 : G$	$(17,22) \leadsto^{\Box} 27$
28)	$a_3 : a_2 : \Diamond_R \neg a_3$	$23 \leadsto^{\downarrow} 28$
29)	$a_4 : \Box_R G$	$(11,24,1,2) \leadsto^{\mathrm{Trans}} 29$

$F = \Diamond_R \top \wedge \mathrm{A}\Box_{R^-} p \wedge \Box_R G$, where $G = {\downarrow}x.\Diamond_R{\downarrow}y.x\colon \Diamond_R \neg y$ and R is transitive

0)	$a_0\colon F$	
1)	$\mathrm{Trans}(R)$	
2)	$R \sqsubseteq R$	
3)	$a_0\colon (\Diamond_R \top \wedge \mathrm{A}\Box_{R^-} p)$	$0 \leadsto^{\wedge} 3$
4)	$a_0\colon \Box_R G$	$0 \leadsto^{\wedge} 4$
5)	$\boxed{a_0\colon \Diamond_R \top}$	$3 \leadsto^{\wedge} 5$
6)	$a_0\colon \mathrm{A}\Box_{R^-} p$	$3 \leadsto^{\wedge} 6$
7)	$a_0\colon \Diamond_R a_1$	$5 \leadsto^{\Diamond} 7$
8)	$a_1\colon \top$	$5 \leadsto^{\Diamond} 8$
9)	$a_1\colon \Box_{R^-} p$	$(6,7) \leadsto^{\mathrm{A}} 9$
10)	$a_0\colon \Box_{R^-} p$	$(6,0) \leadsto^{\mathrm{A}} 10$
11)	$a_1\colon \Box_R G$	$(4,7,1,2) \leadsto^{\mathrm{Trans}} 11$
12)	$a_1\colon G$	$(4,7) \leadsto^{\Box} 12$
13)	$a_0\colon p$	$(9,7) \leadsto^{\Box} 13$
14)	$\boxed{a_1\colon \Diamond_R{\downarrow}y.a_1\colon \Diamond_R \neg y}$	$12 \leadsto^{\downarrow} 14$
15)	$a_1\colon \Diamond_R a_2$	$14 \leadsto^{\Diamond} 15$
16)	$a_2\colon {\downarrow}y.a_1\colon \Diamond_R \neg y$	$14 \leadsto^{\Diamond} 16$
17)	$a_2\colon \Box_R G$	$(11,15,1,2) \leadsto^{\mathrm{Trans}} 17$
18)	$a_2\colon G$	$(11,15) \leadsto^{\Box} 18$
19)	$a_2\colon a_1\colon \Diamond_R \neg a_2$	$16 \leadsto^{\downarrow} 19$
20)	$\boxed{a_2\colon \Diamond_R{\downarrow}y.a_2\colon \Diamond_R \neg y}$	$18 \leadsto^{\downarrow} 20$
21)	$\boxed{a_1\colon \Diamond_R \neg a_2}$	$19 \leadsto^{@} 21$
22)	$a_2\colon \Diamond_R a_3$	$20 \leadsto^{\Diamond} 22$
23)	$a_3\colon {\downarrow}y.a_2\colon \Diamond_R \neg y$	$20 \leadsto^{\Diamond} 23$
24)	$a_1\colon \Diamond_R a_4$	$21 \leadsto^{\Diamond} 24$
25)	$a_4\colon \neg a_2$	$21 \leadsto^{\Diamond} 25$
26)	$a_3\colon \Box_R G$	$(17,22,1,2) \leadsto^{\mathrm{Trans}} 26$
27)	$a_3\colon G$	$(17,22) \leadsto^{\Box} 27$
28)	$a_3\colon a_2\colon \Diamond_R \neg a_3$	$23 \leadsto^{\downarrow} 28$
29)	$a_4\colon \Box_R G$	$(11,24,1,2) \leadsto^{\mathrm{Trans}} 29$
30)	$a_4\colon G$	$(11,24) \leadsto^{\Box} 30$

0)	$a_0 : F$	
1)	$\mathrm{Trans}(R)$	
2)	$R \sqsubseteq R$	
3)	$a_0 : (\Diamond_R \top \wedge \mathrm{A}\Box_{R^-} p)$	$0 \rightsquigarrow^{\wedge} 3$
4)	$a_0 : \Box_R G$	$0 \rightsquigarrow^{\wedge} 4$
5)	$\boxed{a_0 : \Diamond_R \top}$	$3 \rightsquigarrow^{\wedge} 5$
6)	$a_0 : \mathrm{A}\Box_{R^-} p$	$3 \rightsquigarrow^{\wedge} 6$
7)	$a_0 : \Diamond_R a_1$	$5 \rightsquigarrow^{\Diamond} 7$
8)	$a_1 : \top$	$5 \rightsquigarrow^{\Diamond} 8$
9)	$a_1 : \Box_{R^-} p$	$(6,7) \rightsquigarrow^{\mathrm{A}} 9$
10)	$a_0 : \Box_{R^-} p$	$(6,0) \rightsquigarrow^{\mathrm{A}} 10$
11)	$a_1 : \Box_R G$	$(4,7,1,2) \rightsquigarrow^{\mathrm{Trans}} 11$
12)	$a_1 : G$	$(4,7) \rightsquigarrow^{\Box} 12$
13)	$a_0 : p$	$(9,7) \rightsquigarrow^{\Box} 13$
14)	$\boxed{a_1 : \Diamond_R \downarrow y.a_1 : \Diamond_R \neg y}$	$12 \rightsquigarrow^{\downarrow} 14$
15)	$a_1 : \Diamond_R a_2$	$14 \rightsquigarrow^{\Diamond} 15$
16)	$a_2 : \downarrow y.a_1 : \Diamond_R \neg y$	$14 \rightsquigarrow^{\Diamond} 16$
17)	$a_2 : \Box_R G$	$(11,15,1,2) \rightsquigarrow^{\mathrm{Trans}} 17$
18)	$a_2 : G$	$(11,15) \rightsquigarrow^{\Box} 18$
19)	$a_2 : a_1 : \Diamond_R \neg a_2$	$16 \rightsquigarrow^{\downarrow} 19$
20)	$\boxed{a_2 : \Diamond_R \downarrow y.a_2 : \Diamond_R \neg y}$	$18 \rightsquigarrow^{\downarrow} 20$
21)	$\boxed{a_1 : \Diamond_R \neg a_2}$	$19 \rightsquigarrow^{@} 21$
22)	$a_2 : \Diamond_R a_3$	$20 \rightsquigarrow^{\Diamond} 22$
23)	$a_3 : \downarrow y.a_2 : \Diamond_R \neg y$	$20 \rightsquigarrow^{\Diamond} 23$
24)	$a_1 : \Diamond_R a_4$	$21 \rightsquigarrow^{\Diamond} 24$
25)	$a_4 : \neg a_2$	$21 \rightsquigarrow^{\Diamond} 25$
26)	$a_3 : \Box_R G$	$(17,22,1,2) \rightsquigarrow^{\mathrm{Trans}} 26$
27)	$a_3 : G$	$(17,22) \rightsquigarrow^{\Box} 27$
28)	$a_3 : a_2 : \Diamond_R \neg a_3$	$23 \rightsquigarrow^{\downarrow} 28$
29)	$a_4 : \Box_R G$	$(11,24,1,2) \rightsquigarrow^{\mathrm{Trans}} 29$
30)	$a_4 : G$	$(11,24) \rightsquigarrow^{\Box} 30$
31)	$\boxed{a_2 : \Diamond_R \neg a_3}$	$28 \rightsquigarrow^{@} 31$

$F = \Diamond_R \top \wedge \mathsf{A}\Box_{R^-} p \wedge \Box_R G$, where $G = \downarrow x.\Diamond_R \downarrow y.x : \Diamond_R \neg y$ and R is transitive

0)	$a_0 : F$	
1)	$\mathrm{Trans}(R)$	
2)	$R \sqsubseteq R$	
3)	$a_0 : (\Diamond_R \top \wedge \mathsf{A}\Box_{R^-} p)$	$0 \rightsquigarrow^{\wedge} 3$
4)	$a_0 : \Box_R G$	$0 \rightsquigarrow^{\wedge} 4$
5)	$a_0 : \Diamond_R \top$	$3 \rightsquigarrow^{\wedge} 5$
6)	$a_0 : \mathsf{A}\Box_{R^-} p$	$3 \rightsquigarrow^{\wedge} 6$
7)	$a_0 : \Diamond_R a_1$	$5 \rightsquigarrow^{\Diamond} 7$
8)	$a_1 : \top$	$5 \rightsquigarrow^{\Diamond} 8$
9)	$a_1 : \Box_{R^-} p$	$(6,7) \rightsquigarrow^{\mathsf{A}} 9$
10)	$a_0 : \Box_{R^-} p$	$(6,0) \rightsquigarrow^{\mathsf{A}} 10$
11)	$a_1 : \Box_R G$	$(4,7,1,2) \rightsquigarrow^{\mathrm{Trans}} 11$
12)	$a_1 : G$	$(4,7) \rightsquigarrow^{\Box} 12$
13)	$a_0 : p$	$(9,7) \rightsquigarrow^{\Box} 13$
14)	$a_1 : \Diamond_R \downarrow y.a_1 : \Diamond_R \neg y$	$12 \rightsquigarrow^{\downarrow} 14$
15)	$a_1 : \Diamond_R a_2$	$14 \rightsquigarrow^{\Diamond} 15$
16)	$a_2 : \downarrow y.a_1 : \Diamond_R \neg y$	$14 \rightsquigarrow^{\Diamond} 16$
17)	$a_2 : \Box_R G$	$(11,15,1,2) \rightsquigarrow^{\mathrm{Trans}} 17$
18)	$a_2 : G$	$(11,15) \rightsquigarrow^{\Box} 18$
19)	$a_2 : a_1 : \Diamond_R \neg a_2$	$16 \rightsquigarrow^{\downarrow} 19$
20)	$a_2 : \Diamond_R \downarrow y.a_2 : \Diamond_R \neg y$	$18 \rightsquigarrow^{\downarrow} 20$
21)	$a_1 : \Diamond_R \neg a_2$	$19 \rightsquigarrow^{@} 21$
22)	$a_2 : \Diamond_R a_3$	$20 \rightsquigarrow^{\Diamond} 22$
23)	$a_3 : \downarrow y.a_2 : \Diamond_R \neg y$	$20 \rightsquigarrow^{\Diamond} 23$
24)	$a_1 : \Diamond_R a_4$	$21 \rightsquigarrow^{\Diamond} 24$
25)	$a_4 : \neg a_2$	$21 \rightsquigarrow^{\Diamond} 25$
26)	$a_3 : \Box_R G$	$(17,22,1,2) \rightsquigarrow^{\mathrm{Trans}} 26$
27)	$a_3 : G$	$(17,22) \rightsquigarrow^{\Box} 27$
28)	$a_3 : a_2 : \Diamond_R \neg a_3$	$23 \rightsquigarrow^{\downarrow} 28$
29)	$a_4 : \Box_R G$	$(11,24,1,2) \rightsquigarrow^{\mathrm{Trans}} 29$
30)	$a_4 : G$	$(11,24) \rightsquigarrow^{\Box} 30$
31)	$a_2 : \Diamond_R \neg a_3$	$28 \rightsquigarrow^{@} 31$
32)	$a_4 : \Diamond_R \downarrow y.a_4 : \Diamond_R \neg y$	$30 \rightsquigarrow^{\downarrow} 32$

$F = \Diamond_R \top \wedge \mathrm{A}\Box_{R^-} p \wedge \Box_R G$, where $G = \downarrow x.\Diamond_R \downarrow y.x : \Diamond_R \neg y$ and R is transitive

0)	$a_0 : F$	
1)	$\mathrm{Trans}(R)$	
2)	$R \sqsubseteq R$	
3)	$a_0 : (\Diamond_R \top \wedge \mathrm{A}\Box_{R^-} p)$	$0 \leadsto^{\wedge} 3$
4)	$a_0 : \Box_R G$	$0 \leadsto^{\wedge} 4$
5)	$a_0 : \Diamond_R \top$	$3 \leadsto^{\wedge} 5$
6)	$a_0 : \mathrm{A}\Box_{R^-} p$	$3 \leadsto^{\wedge} 6$
7)	$a_0 : \Diamond_R a_1$	$5 \leadsto^{\Diamond} 7$
8)	$a_1 : \top$	$5 \leadsto^{\Diamond} 8$
9)	$a_1 : \Box_{R^-} p$	$(6, 7) \leadsto^{\mathrm{A}} 9$
10)	$a_0 : \Box_{R^-} p$	$(6, 0) \leadsto^{\mathrm{A}} 10$
11)	$a_1 : \Box_R G$	$(4, 7, 1, 2) \leadsto^{\mathrm{Trans}} 11$
12)	$a_1 : G$	$(4, 7) \leadsto^{\Box} 12$
13)	$a_0 : p$	$(9, 7) \leadsto^{\Box} 13$
14)	$a_1 : \Diamond_R \downarrow y.a_1 : \Diamond_R \neg y$	$12 \leadsto^{\downarrow} 14$
15)	$a_1 : \Diamond_R a_2$	$14 \leadsto^{\Diamond} 15$
16)	$a_2 : \downarrow y.a_1 : \Diamond_R \neg y$	$14 \leadsto^{\Diamond} 16$
17)	$a_2 : \Box_R G$	$(11, 15, 1, 2) \leadsto^{\mathrm{Trans}} 17$
18)	$a_2 : G$	$(11, 15) \leadsto^{\Box} 18$
19)	$a_2 : a_1 : \Diamond_R \neg a_2$	$16 \leadsto^{\downarrow} 19$
20)	$a_2 : \Diamond_R \downarrow y.a_2 : \Diamond_R \neg y$	$18 \leadsto^{\downarrow} 20$
21)	$a_1 : \Diamond_R \neg a_2$	$19 \leadsto^{@} 21$
22)	$a_2 : \Diamond_R a_3$	$20 \leadsto^{\Diamond} 22$
23)	$a_3 : \downarrow y.a_2 : \Diamond_R \neg y$	$20 \leadsto^{\Diamond} 23$
24)	$a_1 : \Diamond_R a_4$	$21 \leadsto^{\Diamond} 24$
25)	$a_4 : \neg a_2$	$21 \leadsto^{\Diamond} 25$
26)	$a_3 : \Box_R G$	$(17, 22, 1, 2) \leadsto^{\mathrm{Trans}} 26$
27)	$a_3 : G$	$(17, 22) \leadsto^{\Box} 27$
28)	$a_3 : a_2 : \Diamond_R \neg a_3$	$23 \leadsto^{\downarrow} 28$
29)	$a_4 : \Box_R G$	$(11, 24, 1, 2) \leadsto^{\mathrm{Trans}} 29$
30)	$a_4 : G$	$(11, 24) \leadsto^{\Box} 30$
31)	$a_2 : \Diamond_R \neg a_3$	$28 \leadsto^{@} 31$
32)	$a_4 : \Diamond_R \downarrow y.a_4 : \Diamond_R \neg y$	$30 \leadsto^{\downarrow} 32$
33)	$a_2 : \Diamond_R a_5$	$31 \leadsto^{\Diamond} 33$
34)	$a_5 : \neg a_3$	$31 \leadsto^{\Diamond} 34$

$F = \Diamond_R \top \wedge \mathrm{A}\Box_{R^-} p \wedge \Box_R G$, where $G = \downarrow x.\Diamond_R \downarrow y.x{:}\,\Diamond_R \neg y$ and R is transitive

0)	$a_0{:}\,F$	
1)	$\mathrm{Trans}(R)$	
2)	$R \sqsubseteq R$	
3)	$a_0{:}\,(\Diamond_R \top \wedge \mathrm{A}\Box_{R^-} p)$	$0 \leadsto^{\wedge} 3$
4)	$a_0{:}\,\Box_R G$	$0 \leadsto^{\wedge} 4$
5)	$a_0{:}\,\Diamond_R \top$	$3 \leadsto^{\wedge} 5$
6)	$a_0{:}\,\mathrm{A}\Box_{R^-} p$	$3 \leadsto^{\wedge} 6$
7)	$a_0{:}\,\Diamond_R a_1$	$5 \leadsto^{\Diamond} 7$
8)	$a_1{:}\,\top$	$5 \leadsto^{\Diamond} 8$
9)	$a_1{:}\,\Box_{R^-} p$	$(6,7) \leadsto^{\mathrm{A}} 9$
10)	$a_0{:}\,\Box_{R^-} p$	$(6,0) \leadsto^{\mathrm{A}} 10$
11)	$a_1{:}\,\Box_R G$	$(4,7,1,2) \leadsto^{\mathrm{Trans}} 11$
12)	$a_1{:}\,G$	$(4,7) \leadsto^{\Box} 12$
13)	$a_0{:}\,p$	$(9,7) \leadsto^{\Box} 13$
14)	$a_1{:}\,\Diamond_R \downarrow y.a_1{:}\,\Diamond_R \neg y$	$12 \leadsto^{\downarrow} 14$
15)	$a_1{:}\,\Diamond_R a_2$	$14 \leadsto^{\Diamond} 15$
16)	$a_2{:}\,\downarrow y.a_1{:}\,\Diamond_R \neg y$	$14 \leadsto^{\Diamond} 16$
17)	$a_2{:}\,\Box_R G$	$(11,15,1,2) \leadsto^{\mathrm{Trans}} 17$
18)	$a_2{:}\,G$	$(11,15) \leadsto^{\Box} 18$
19)	$a_2{:}\,a_1{:}\,\Diamond_R \neg a_2$	$16 \leadsto^{\downarrow} 19$
20)	$a_2{:}\,\Diamond_R \downarrow y.a_2{:}\,\Diamond_R \neg y$	$18 \leadsto^{\downarrow} 20$
21)	$a_1{:}\,\Diamond_R \neg a_2$	$19 \leadsto^{@} 21$
22)	$a_2{:}\,\Diamond_R a_3$	$20 \leadsto^{\Diamond} 22$
23)	$a_3{:}\,\downarrow y.a_2{:}\,\Diamond_R \neg y$	$20 \leadsto^{\Diamond} 23$
24)	$a_1{:}\,\Diamond_R a_4$	$21 \leadsto^{\Diamond} 24$
25)	$a_4{:}\,\neg a_2$	$21 \leadsto^{\Diamond} 25$
26)	$a_3{:}\,\Box_R G$	$(17,22,1,2) \leadsto^{\mathrm{Trans}} 26$
27)	$a_3{:}\,G$	$(17,22) \leadsto^{\Box} 27$
28)	$a_3{:}\,a_2{:}\,\Diamond_R \neg a_3$	$23 \leadsto^{\downarrow} 28$
29)	$a_4{:}\,\Box_R G$	$(11,24,1,2) \leadsto^{\mathrm{Trans}} 29$
30)	$a_4{:}\,G$	$(11,24) \leadsto^{\Box} 30$
31)	$a_2{:}\,\Diamond_R \neg a_3$	$28 \leadsto^{@} 31$
32)	$a_4{:}\,\Diamond_R \downarrow y.a_4{:}\,\Diamond_R \neg y$	$30 \leadsto^{\downarrow} 32$
33)	$a_2{:}\,\Diamond_R a_5$	$31 \leadsto^{\Diamond} 33$
34)	$a_5{:}\,\neg a_3$	$31 \leadsto^{\Diamond} 34$
35)	$a_4{:}\,\Box_{R^-} p$	$(6,24) \leadsto^{\mathrm{A}} 35$

$F = \Diamond_R \top \wedge \mathsf{A}\Box_{R^-} p \wedge \Box_R G$, where $G = \downarrow x.\Diamond_R \downarrow y.x: \Diamond_R \neg y$ and R is transitive

0)	$a_0 : F$	
1)	$\mathrm{Trans}(R)$	
2)	$R \sqsubseteq R$	
3)	$a_0 : (\Diamond_R \top \wedge \mathsf{A}\Box_{R^-} p)$	$0 \leadsto^{\wedge} 3$
4)	$a_0 : \Box_R G$	$0 \leadsto^{\wedge} 4$
5)	$a_0 : \Diamond_R \top$	$3 \leadsto^{\wedge} 5$
6)	$a_0 : \mathsf{A}\Box_{R^-} p$	$3 \leadsto^{\wedge} 6$
7)	$a_0 : \Diamond_R a_1$	$5 \leadsto^{\Diamond} 7$
8)	$a_1 : \top$	$5 \leadsto^{\Diamond} 8$
9)	$a_1 : \Box_{R^-} p$	$(6, 7) \leadsto^{\mathsf{A}} 9$
10)	$a_0 : \Box_{R^-} p$	$(6, 0) \leadsto^{\mathsf{A}} 10$
11)	$a_1 : \Box_R G$	$(4, 7, 1, 2) \leadsto^{\mathrm{Trans}} 11$
12)	$a_1 : G$	$(4, 7) \leadsto^{\Box} 12$
13)	$a_0 : p$	$(9, 7) \leadsto^{\Box} 13$
14)	$a_1 : \Diamond_R \downarrow y.a_1 : \Diamond_R \neg y$	$12 \leadsto^{\downarrow} 14$
15)	$a_1 : \Diamond_R a_2$	$14 \leadsto^{\Diamond} 15$
16)	$a_2 : \downarrow y.a_1 : \Diamond_R \neg y$	$14 \leadsto^{\Diamond} 16$
17)	$a_2 : \Box_R G$	$(11, 15, 1, 2) \leadsto^{\mathrm{Trans}} 17$
18)	$a_2 : G$	$(11, 15) \leadsto^{\Box} 18$
19)	$a_2 : a_1 : \Diamond_R \neg a_2$	$16 \leadsto^{\downarrow} 19$
20)	$a_2 : \Diamond_R \downarrow y.a_2 : \Diamond_R \neg y$	$18 \leadsto^{\downarrow} 20$
21)	$a_1 : \Diamond_R \neg a_2$	$19 \leadsto^{@} 21$
22)	$a_2 : \Diamond_R a_3$	$20 \leadsto^{\Diamond} 22$
23)	$a_3 : \downarrow y.a_2 : \Diamond_R \neg y$	$20 \leadsto^{\Diamond} 23$
24)	$a_1 : \Diamond_R a_4$	$21 \leadsto^{\Diamond} 24$
25)	$a_4 : \neg a_2$	$21 \leadsto^{\Diamond} 25$
26)	$a_3 : \Box_R G$	$(17, 22, 1, 2) \leadsto^{\mathrm{Trans}} 26$
27)	$a_3 : G$	$(17, 22) \leadsto^{\Box} 27$
28)	$a_3 : a_2 : \Diamond_R \neg a_3$	$23 \leadsto^{\downarrow} 28$
29)	$a_4 : \Box_R G$	$(11, 24, 1, 2) \leadsto^{\mathrm{Trans}} 29$
30)	$a_4 : G$	$(11, 24) \leadsto^{\Box} 30$
31)	$a_2 : \Diamond_R \neg a_3$	$28 \leadsto^{@} 31$
32)	$a_4 : \Diamond_R \downarrow y.a_4 : \Diamond_R \neg y$	$30 \leadsto^{\downarrow} 32$
33)	$a_2 : \Diamond_R a_5$	$31 \leadsto^{\Diamond} 33$
34)	$a_5 : \neg a_3$	$31 \leadsto^{\Diamond} 34$
35)	$a_4 : \Box_{R^-} p$	$(6, 24) \leadsto^{\mathsf{A}} 35$
36)	$a_3 : \Box_{R^-} p$	$(6, 22) \leadsto^{\mathsf{A}} 36$

$F = \Diamond_R \top \wedge \mathsf{A}\Box_{R^-} p \wedge \Box_R G$, where $G = \downarrow x.\Diamond_R \downarrow y.x{:}\,\Diamond_R \neg y$ and R is transitive

0)	$a_0{:}\, F$	
1)	$\mathrm{Trans}(R)$	
2)	$R \sqsubseteq R$	
3)	$a_0{:}\,(\Diamond_R \top \wedge \mathsf{A}\Box_{R^-} p)$	$0 \rightsquigarrow^{\wedge} 3$
4)	$a_0{:}\,\Box_R G$	$0 \rightsquigarrow^{\wedge} 4$
5)	$a_0{:}\,\Diamond_R \top$	$3 \rightsquigarrow^{\wedge} 5$
6)	$a_0{:}\,\mathsf{A}\Box_{R^-} p$	$3 \rightsquigarrow^{\wedge} 6$
7)	$a_0{:}\,\Diamond_R a_1$	$5 \rightsquigarrow^{\Diamond} 7$
8)	$a_1{:}\,\top$	$5 \rightsquigarrow^{\Diamond} 8$
9)	$a_1{:}\,\Box_{R^-} p$	$(6,7) \rightsquigarrow^{\mathsf{A}} 9$
10)	$a_0{:}\,\Box_{R^-} p$	$(6,0) \rightsquigarrow^{\mathsf{A}} 10$
11)	$a_1{:}\,\Box_R G$	$(4,7,1,2) \rightsquigarrow^{\mathrm{Trans}} 11$
12)	$a_1{:}\,G$	$(4,7) \rightsquigarrow^{\Box} 12$
13)	$a_0{:}\,p$	$(9,7) \rightsquigarrow^{\Box} 13$
14)	$a_1{:}\,\Diamond_R \downarrow y.a_1{:}\,\Diamond_R \neg y$	$12 \rightsquigarrow^{\downarrow} 14$
15)	$a_1{:}\,\Diamond_R a_2$	$14 \rightsquigarrow^{\Diamond} 15$
16)	$a_2{:}\,\downarrow y.a_1{:}\,\Diamond_R \neg y$	$14 \rightsquigarrow^{\Diamond} 16$
17)	$a_2{:}\,\Box_R G$	$(11,15,1,2) \rightsquigarrow^{\mathrm{Trans}} 17$
18)	$a_2{:}\,G$	$(11,15) \rightsquigarrow^{\Box} 18$
19)	$a_2{:}\,a_1{:}\,\Diamond_R \neg a_2$	$16 \rightsquigarrow^{\downarrow} 19$
20)	$a_2{:}\,\Diamond_R \downarrow y.a_2{:}\,\Diamond_R \neg y$	$18 \rightsquigarrow^{\downarrow} 20$
21)	$a_1{:}\,\Diamond_R \neg a_2$	$19 \rightsquigarrow^{@} 21$
22)	$a_2{:}\,\Diamond_R a_3$	$20 \rightsquigarrow^{\Diamond} 22$
23)	$a_3{:}\,\downarrow y.a_2{:}\,\Diamond_R \neg y$	$20 \rightsquigarrow^{\Diamond} 23$
24)	$a_1{:}\,\Diamond_R a_4$	$21 \rightsquigarrow^{\Diamond} 24$
25)	$a_4{:}\,\neg a_2$	$21 \rightsquigarrow^{\Diamond} 25$
26)	$a_3{:}\,\Box_R G$	$(17,22,1,2) \rightsquigarrow^{\mathrm{Trans}} 26$
27)	$a_3{:}\,G$	$(17,22) \rightsquigarrow^{\Box} 27$
28)	$a_3{:}\,a_2{:}\,\Diamond_R \neg a_3$	$23 \rightsquigarrow^{\downarrow} 28$
29)	$a_4{:}\,\Box_R G$	$(11,24,1,2) \rightsquigarrow^{\mathrm{Trans}} 29$
30)	$a_4{:}\,G$	$(11,24) \rightsquigarrow^{\Box} 30$
31)	$a_2{:}\,\Diamond_R \neg a_3$	$28 \rightsquigarrow^{@} 31$
32)	$a_4{:}\,\Diamond_R \downarrow y.a_4{:}\,\Diamond_R \neg y$	$30 \rightsquigarrow^{\downarrow} 32$
33)	$a_2{:}\,\Diamond_R a_5$	$31 \rightsquigarrow^{\Diamond} 33$
34)	$a_5{:}\,\neg a_3$	$31 \rightsquigarrow^{\Diamond} 34$
35)	$a_4{:}\,\Box_{R^-} p$	$(6,24) \rightsquigarrow^{\mathsf{A}} 35$
36)	$a_3{:}\,\Box_{R^-} p$	$(6,22) \rightsquigarrow^{\mathsf{A}} 36$
37)	$a_2{:}\,\Box_{R^-} p$	$(6,15) \rightsquigarrow^{\mathsf{A}} 37$

$F = \Diamond_R \top \wedge \mathsf{A}\Box_{R^-} p \wedge \Box_R G$, where $G = {\downarrow} x.\Diamond_R {\downarrow} y.x : \Diamond_R \neg y$ and R is transitive

0)	$a_0 : F$	
1)	Trans(R)	
2)	$R \sqsubseteq R$	
3)	$a_0 : (\Diamond_R \top \wedge \mathsf{A}\Box_{R^-} p)$	$0 \rightsquigarrow^{\wedge} 3$
4)	$a_0 : \Box_R G$	$0 \rightsquigarrow^{\wedge} 4$
5)	$a_0 : \Diamond_R \top$	$3 \rightsquigarrow^{\wedge} 5$
6)	$a_0 : \mathsf{A}\Box_{R^-} p$	$3 \rightsquigarrow^{\wedge} 6$
7)	$a_0 : \Diamond_R a_1$	$5 \rightsquigarrow^{\Diamond} 7$
8)	$a_1 : \top$	$5 \rightsquigarrow^{\Diamond} 8$
9)	$a_1 : \Box_{R^-} p$	$(6, 7) \rightsquigarrow^{\mathsf{A}} 9$
10)	$a_0 : \Box_{R^-} p$	$(6, 0) \rightsquigarrow^{\mathsf{A}} 10$
11)	$a_1 : \Box_R G$	$(4, 7, 1, 2) \rightsquigarrow^{\mathrm{Trans}} 11$
12)	$a_1 : G$	$(4, 7) \rightsquigarrow^{\Box} 12$
13)	$a_0 : p$	$(9, 7) \rightsquigarrow^{\Box} 13$
14)	$a_1 : \Diamond_R {\downarrow} y.a_1 : \Diamond_R \neg y$	$12 \rightsquigarrow^{\downarrow} 14$
15)	$a_1 : \Diamond_R a_2$	$14 \rightsquigarrow^{\Diamond} 15$
16)	$a_2 : {\downarrow} y.a_1 : \Diamond_R \neg y$	$14 \rightsquigarrow^{\Diamond} 16$
17)	$a_2 : \Box_R G$	$(11, 15, 1, 2) \rightsquigarrow^{\mathrm{Trans}} 17$
18)	$a_2 : G$	$(11, 15) \rightsquigarrow^{\Box} 18$
19)	$a_2 : a_1 : \Diamond_R \neg a_2$	$16 \rightsquigarrow^{\downarrow} 19$
20)	$a_2 : \Diamond_R {\downarrow} y.a_2 : \Diamond_R \neg y$	$18 \rightsquigarrow^{\downarrow} 20$
21)	$a_1 : \Diamond_R \neg a_2$	$19 \rightsquigarrow^{@} 21$
22)	$a_2 : \Diamond_R a_3$	$20 \rightsquigarrow^{\Diamond} 22$
23)	$a_3 : {\downarrow} y.a_2 : \Diamond_R \neg y$	$20 \rightsquigarrow^{\Diamond} 23$
24)	$a_1 : \Diamond_R a_4$	$21 \rightsquigarrow^{\Diamond} 24$
25)	$a_4 : \neg a_2$	$21 \rightsquigarrow^{\Diamond} 25$
26)	$a_3 : \Box_R G$	$(17, 22, 1, 2) \rightsquigarrow^{\mathrm{Trans}} 26$
27)	$a_3 : G$	$(17, 22) \rightsquigarrow^{\Box} 27$
28)	$a_3 : a_2 : \Diamond_R \neg a_3$	$23 \rightsquigarrow^{\downarrow} 28$
29)	$a_4 : \Box_R G$	$(11, 24, 1, 2) \rightsquigarrow^{\mathrm{Trans}} 29$
30)	$a_4 : G$	$(11, 24) \rightsquigarrow^{\Box} 30$
31)	$a_2 : \Diamond_R \neg a_3$	$28 \rightsquigarrow^{@} 31$
32)	$a_4 : \Diamond_R {\downarrow} y.a_4 : \Diamond_R \neg y$	$30 \rightsquigarrow^{\downarrow} 32$
33)	$a_2 : \Diamond_R a_5$	$31 \rightsquigarrow^{\Diamond} 33$
34)	$a_5 : \neg a_3$	$31 \rightsquigarrow^{\Diamond} 34$
35)	$a_4 : \Box_{R^-} p$	$(6, 24) \rightsquigarrow^{\mathsf{A}} 35$
36)	$a_3 : \Box_{R^-} p$	$(6, 22) \rightsquigarrow^{\mathsf{A}} 36$
37)	$a_2 : \Box_{R^-} p$	$(6, 15) \rightsquigarrow^{\mathsf{A}} 37$
38)	$a_1 : p$	$(35, 24) \rightsquigarrow^{\Box} 38$

$F = \Diamond_R \top \wedge \mathsf{A}\Box_{R^-} p \wedge \Box_R G$, where $G = \downarrow x.\Diamond_R \downarrow y.x : \Diamond_R \neg y$ and R is transitive

0)	$a_0 : F$	
1)	$\mathrm{Trans}(R)$	
2)	$R \sqsubseteq R$	
3)	$a_0 : (\Diamond_R \top \wedge \mathsf{A}\Box_{R^-} p)$	$0 \leadsto^{\wedge} 3$
4)	$a_0 : \Box_R G$	$0 \leadsto^{\wedge} 4$
5)	$a_0 : \Diamond_R \top$	$3 \leadsto^{\wedge} 5$
6)	$a_0 : \mathsf{A}\Box_{R^-} p$	$3 \leadsto^{\wedge} 6$
7)	$a_0 : \Diamond_R a_1$	$5 \leadsto^{\Diamond} 7$
8)	$a_1 : \top$	$5 \leadsto^{\Diamond} 8$
9)	$a_1 : \Box_{R^-} p$	$(6,7) \leadsto^{\mathsf{A}} 9$
10)	$a_0 : \Box_{R^-} p$	$(6,0) \leadsto^{\mathsf{A}} 10$
11)	$a_1 : \Box_R G$	$(4,7,1,2) \leadsto^{\mathrm{Trans}} 11$
12)	$a_1 : G$	$(4,7) \leadsto^{\Box} 12$
13)	$a_0 : p$	$(9,7) \leadsto^{\Box} 13$
14)	$a_1 : \Diamond_R \downarrow y.a_1 : \Diamond_R \neg y$	$12 \leadsto^{\downarrow} 14$
15)	$a_1 : \Diamond_R a_2$	$14 \leadsto^{\Diamond} 15$
16)	$a_2 : \downarrow y.a_1 : \Diamond_R \neg y$	$14 \leadsto^{\Diamond} 16$
17)	$a_2 : \Box_R G$	$(11,15,1,2) \leadsto^{\mathrm{Trans}} 17$
18)	$a_2 : G$	$(11,15) \leadsto^{\Box} 18$
19)	$a_2 : a_1 : \Diamond_R \neg a_2$	$16 \leadsto^{\downarrow} 19$
20)	$a_2 : \Diamond_R \downarrow y.a_2 : \Diamond_R \neg y$	$18 \leadsto^{\downarrow} 20$
21)	$a_1 : \Diamond_R \neg a_2$	$19 \leadsto^{@} 21$

22)	$a_2 : \Diamond_R a_3$	$20 \leadsto^{\Diamond} 22$
23)	$a_3 : \downarrow y.a_2 : \Diamond_R \neg y$	$20 \leadsto^{\Diamond} 23$
24)	$a_1 : \Diamond_R a_4$	$21 \leadsto^{\Diamond} 24$
25)	$a_4 : \neg a_2$	$21 \leadsto^{\Diamond} 25$
26)	$a_3 : \Box_R G$	$(17,22,1,2) \leadsto^{\mathrm{Trans}} 26$
27)	$a_3 : G$	$(17,22) \leadsto^{\Box} 27$
28)	$a_3 : a_2 : \Diamond_R \neg a_3$	$23 \leadsto^{\downarrow} 28$
29)	$a_4 : \Box_R G$	$(11,24,1,2) \leadsto^{\mathrm{Trans}} 29$
30)	$a_4 : G$	$(11,24) \leadsto^{\Box} 30$
31)	$a_2 : \Diamond_R \neg a_3$	$28 \leadsto^{@} 31$
32)	$a_4 : \Diamond_R \downarrow y.a_4 : \Diamond_R \neg y$	$30 \leadsto^{\downarrow} 32$
33)	$a_2 : \Diamond_R a_5$	$31 \leadsto^{\Diamond} 33$
34)	$a_5 : \neg a_3$	$31 \leadsto^{\Diamond} 34$
35)	$a_4 : \Box_{R^-} p$	$(6,24) \leadsto^{\mathsf{A}} 35$
36)	$a_3 : \Box_{R^-} p$	$(6,22) \leadsto^{\mathsf{A}} 36$
37)	$a_2 : \Box_{R^-} p$	$(6,15) \leadsto^{\mathsf{A}} 37$
38)	$a_1 : p$	$(35,24) \leadsto^{\Box} 38$
39)	$a_2 : p$	$(36,22) \leadsto^{\Box} 39$

$F = \Diamond_R \top \wedge \mathsf{A}\Box_{R^-} p \wedge \Box_R G$, where $G = \downarrow x.\Diamond_R \downarrow y.x{:}\Diamond_R \neg y$ and R is transitive

0)	$a_0 : F$	
1)	$\mathrm{Trans}(R)$	
2)	$R \sqsubseteq R$	
3)	$a_0 : (\Diamond_R \top \wedge \mathsf{A}\Box_{R^-} p)$	$0 \leadsto^{\wedge} 3$
4)	$a_0 : \Box_R G$	$0 \leadsto^{\wedge} 4$
5)	$a_0 : \Diamond_R \top$	$3 \leadsto^{\wedge} 5$
6)	$a_0 : \mathsf{A}\Box_{R^-} p$	$3 \leadsto^{\wedge} 6$
7)	$a_0 : \Diamond_R a_1$	$5 \leadsto^{\Diamond} 7$
8)	$a_1 : \top$	$5 \leadsto^{\Diamond} 8$
9)	$a_1 : \Box_{R^-} p$	$(6,7) \leadsto^{\mathsf{A}} 9$
10)	$a_0 : \Box_{R^-} p$	$(6,0) \leadsto^{\mathsf{A}} 10$
11)	$a_1 : \Box_R G$	$(4,7,1,2) \leadsto^{\mathrm{Trans}} 11$
12)	$a_1 : G$	$(4,7) \leadsto^{\Box} 12$
13)	$a_0 : p$	$(9,7) \leadsto^{\Box} 13$
14)	$a_1 : \Diamond_R \downarrow y.a_1 : \Diamond_R \neg y$	$12 \leadsto^{\downarrow} 14$
15)	$a_1 : \Diamond_R a_2$	$14 \leadsto^{\Diamond} 15$
16)	$a_2 : \downarrow y.a_1 : \Diamond_R \neg y$	$14 \leadsto^{\Diamond} 16$
17)	$a_2 : \Box_R G$	$(11,15,1,2) \leadsto^{\mathrm{Trans}} 17$
18)	$a_2 : G$	$(11,15) \leadsto^{\Box} 18$
19)	$a_2 : a_1 : \Diamond_R \neg a_2$	$16 \leadsto^{\downarrow} 19$
20)	$a_2 : \Diamond_R \downarrow y.a_2 : \Diamond_R \neg y$	$18 \leadsto^{\downarrow} 20$
21)	$a_1 : \Diamond_R \neg a_2$	$19 \leadsto^{@} 21$
22)	$a_2 : \Diamond_R a_3$	$20 \leadsto^{\Diamond} 22$
23)	$a_3 : \downarrow y.a_2 : \Diamond_R \neg y$	$20 \leadsto^{\Diamond} 23$
24)	$a_1 : \Diamond_R a_4$	$21 \leadsto^{\Diamond} 24$
25)	$a_4 : \neg a_2$	$21 \leadsto^{\Diamond} 25$
26)	$a_3 : \Box_R G$	$(17,22,1,2) \leadsto^{\mathrm{Trans}} 26$
27)	$a_3 : G$	$(17,22) \leadsto^{\Box} 27$
28)	$a_3 : a_2 : \Diamond_R \neg a_3$	$23 \leadsto^{\downarrow} 28$
29)	$a_4 : \Box_R G$	$(11,24,1,2) \leadsto^{\mathrm{Trans}} 29$
30)	$a_4 : G$	$(11,24) \leadsto^{\Box} 30$
31)	$a_2 : \Diamond_R \neg a_3$	$28 \leadsto^{@} 31$
32)	$a_4 : \Diamond_R \downarrow y.a_4 : \Diamond_R \neg y$	$30 \leadsto^{\downarrow} 32$
33)	$a_2 : \Diamond_R a_5$	$31 \leadsto^{\Diamond} 33$
34)	$a_5 : \neg a_3$	$31 \leadsto^{\Diamond} 34$
35)	$a_4 : \Box_{R^-} p$	$(6,24) \leadsto^{\mathsf{A}} 35$
36)	$a_3 : \Box_{R^-} p$	$(6,22) \leadsto^{\mathsf{A}} 36$
37)	$a_2 : \Box_{R^-} p$	$(6,15) \leadsto^{\mathsf{A}} 37$
38)	$a_1 : p$	$(35,24) \leadsto^{\Box} 38$
39)	$a_2 : p$	$(36,22) \leadsto^{\Box} 39$
40)	$a_4 : \Diamond_R a_6$	$32 \leadsto^{\Diamond} 40$
41)	$a_6 : \downarrow y.a_4 : \Diamond_R \neg y$	$32 \leadsto^{\Diamond} 41$

$F = \Diamond_R \top \wedge \mathsf{A}\Box_{R^-} p \wedge \Box_R G$, where $G = \downarrow x.\Diamond_R \downarrow y.x\colon \Diamond_R \neg y$ and R is transitive

0)	$a_0\colon F$	
1)	$\mathrm{Trans}(R)$	
2)	$R \sqsubseteq R$	
3)	$a_0\colon (\Diamond_R \top \wedge \mathsf{A}\Box_{R^-} p)$	$0 \rightsquigarrow^{\wedge} 3$
4)	$a_0\colon \Box_R G$	$0 \rightsquigarrow^{\wedge} 4$
5)	$a_0\colon \Diamond_R \top$	$3 \rightsquigarrow^{\wedge} 5$
6)	$a_0\colon \mathsf{A}\Box_{R^-} p$	$3 \rightsquigarrow^{\wedge} 6$
7)	$a_0\colon \Diamond_R a_1$	$5 \rightsquigarrow^{\Diamond} 7$
8)	$a_1\colon \top$	$5 \rightsquigarrow^{\Diamond} 8$
9)	$a_1\colon \Box_{R^-} p$	$(6,7) \rightsquigarrow^{\mathsf{A}} 9$
10)	$a_0\colon \Box_{R^-} p$	$(6,0) \rightsquigarrow^{\mathsf{A}} 10$
11)	$a_1\colon \Box_R G$	$(4,7,1,2) \rightsquigarrow^{\mathrm{Trans}} 11$
12)	$a_1\colon G$	$(4,7) \rightsquigarrow^{\Box} 12$
13)	$a_0\colon p$	$(9,7) \rightsquigarrow^{\Box} 13$
14)	$a_1\colon \Diamond_R \downarrow y.a_1\colon \Diamond_R \neg y$	$12 \rightsquigarrow^{\downarrow} 14$
15)	$a_1\colon \Diamond_R a_2$	$14 \rightsquigarrow^{\Diamond} 15$
16)	$a_2\colon \downarrow y.a_1\colon \Diamond_R \neg y$	$14 \rightsquigarrow^{\Diamond} 16$
17)	$a_2\colon \Box_R G$	$(11,15,1,2) \rightsquigarrow^{\mathrm{Trans}} 17$
18)	$a_2\colon G$	$(11,15) \rightsquigarrow^{\Box} 18$
19)	$a_2\colon a_1\colon \Diamond_R \neg a_2$	$16 \rightsquigarrow^{\downarrow} 19$
20)	$a_2\colon \Diamond_R \downarrow y.a_2\colon \Diamond_R \neg y$	$18 \rightsquigarrow^{\downarrow} 20$
21)	$a_1\colon \Diamond_R \neg a_2$	$19 \rightsquigarrow^{@} 21$
22)	$a_2\colon \Diamond_R a_3$	$20 \rightsquigarrow^{\Diamond} 22$
23)	$a_3\colon \downarrow y.a_2\colon \Diamond_R \neg y$	$20 \rightsquigarrow^{\Diamond} 23$
24)	$a_1\colon \Diamond_R a_4$	$21 \rightsquigarrow^{\Diamond} 24$
25)	$a_4\colon \neg a_2$	$21 \rightsquigarrow^{\Diamond} 25$
26)	$a_3\colon \Box_R G$	$(17,22,1,2) \rightsquigarrow^{\mathrm{Trans}} 26$
27)	$a_3\colon G$	$(17,22) \rightsquigarrow^{\Box} 27$
28)	$a_3\colon a_2\colon \Diamond_R \neg a_3$	$23 \rightsquigarrow^{\downarrow} 28$
29)	$a_4\colon \Box_R G$	$(11,24,1,2) \rightsquigarrow^{\mathrm{Trans}} 29$
30)	$a_4\colon G$	$(11,24) \rightsquigarrow^{\Box} 30$
31)	$a_2\colon \Diamond_R \neg a_3$	$28 \rightsquigarrow^{@} 31$
32)	$a_4\colon \Diamond_R \downarrow y.a_4\colon \Diamond_R \neg y$	$30 \rightsquigarrow^{\downarrow} 32$
33)	$a_2\colon \Diamond_R a_5$	$31 \rightsquigarrow^{\Diamond} 33$
34)	$a_5\colon \neg a_3$	$31 \rightsquigarrow^{\Diamond} 34$
35)	$a_4\colon \Box_{R^-} p$	$(6,24) \rightsquigarrow^{\mathsf{A}} 35$
36)	$a_3\colon \Box_{R^-} p$	$(6,22) \rightsquigarrow^{\mathsf{A}} 36$
37)	$a_2\colon \Box_{R^-} p$	$(6,15) \rightsquigarrow^{\mathsf{A}} 37$
38)	$a_1\colon p$	$(35,24) \rightsquigarrow^{\Box} 38$
39)	$a_2\colon p$	$(36,22) \rightsquigarrow^{\Box} 39$
40)	$a_4\colon \Diamond_R a_6$	$32 \rightsquigarrow^{\Diamond} 40$
41)	$a_6\colon \downarrow y.a_4\colon \Diamond_R \neg y$	$32 \rightsquigarrow^{\Diamond} 41$
42)	$a_6\colon \Box_{R^-} p$	$(6,40) \rightsquigarrow^{\mathsf{A}} 42$

$F = \Diamond_R\top \wedge \mathsf{A}\Box_{R^-}p \wedge \Box_R G$, where $G = \downarrow x.\Diamond_R\downarrow y.x\colon \Diamond_R\neg y$ and R is transitive

0)	$a_0\colon F$	
1)	$\mathrm{Trans}(R)$	
2)	$R \sqsubseteq R$	
3)	$a_0\colon (\Diamond_R\top \wedge \mathsf{A}\Box_{R^-}p)$	$0 \leadsto^{\wedge} 3$
4)	$a_0\colon \Box_R G$	$0 \leadsto^{\wedge} 4$
5)	$\boxed{a_0\colon \Diamond_R\top}$	$3 \leadsto^{\wedge} 5$
6)	$a_0\colon \mathsf{A}\Box_{R^-}p$	$3 \leadsto^{\wedge} 6$
7)	$a_0\colon \Diamond_R a_1$	$5 \leadsto^{\Diamond} 7$
8)	$a_1\colon \top$	$5 \leadsto^{\Diamond} 8$
9)	$a_1\colon \Box_{R^-}p$	$(6,7) \leadsto^{\mathsf{A}} 9$
10)	$a_0\colon \Box_{R^-}p$	$(6,0) \leadsto^{\mathsf{A}} 10$
11)	$a_1\colon \Box_R G$	$(4,7,1,2) \leadsto^{\mathrm{Trans}} 11$
12)	$a_1\colon G$	$(4,7) \leadsto^{\Box} 12$
13)	$a_0\colon p$	$(9,7) \leadsto^{\Box} 13$
14)	$a_1\colon \Diamond_R\downarrow y.a_1\colon \Diamond_R\neg y$	$12 \leadsto^{\downarrow} 14$
15)	$a_1\colon \Diamond_R a_2$	$14 \leadsto^{\Diamond} 15$
16)	$a_2\colon \downarrow y.a_1\colon \Diamond_R\neg y$	$14 \leadsto^{\Diamond} 16$
17)	$a_2\colon \Box_R G$	$(11,15,1,2) \leadsto^{\mathrm{Trans}} 17$
18)	$a_2\colon G$	$(11,15) \leadsto^{\Box} 18$
19)	$a_2\colon a_1\colon \Diamond_R\neg a_2$	$16 \leadsto^{\downarrow} 19$
20)	$a_2\colon \Diamond_R\downarrow y.a_2\colon \Diamond_R\neg y$	$18 \leadsto^{\downarrow} 20$
21)	$\boxed{a_1\colon \Diamond_R\neg a_2}$	$19 \leadsto^{@} 21$

22)	$a_2\colon \Diamond_R a_3$	$20 \leadsto^{\Diamond} 22$
23)	$a_3\colon \downarrow y.a_2\colon \Diamond_R\neg y$	$20 \leadsto^{\Diamond} 23$
24)	$a_1\colon \Diamond_R a_4$	$21 \leadsto^{\Diamond} 24$
25)	$a_4\colon \neg a_2$	$21 \leadsto^{\Diamond} 25$
26)	$a_3\colon \Box_R G$	$(17,22,1,2) \leadsto^{\mathrm{Trans}} 26$
27)	$a_3\colon G$	$(17,22) \leadsto^{\Box} 27$
28)	$a_3\colon a_2\colon \Diamond_R\neg a_3$	$23 \leadsto^{\downarrow} 28$
29)	$a_4\colon \Box_R G$	$(11,24,1,2) \leadsto^{\mathrm{Trans}} 29$
30)	$a_4\colon G$	$(11,24) \leadsto^{\Box} 30$
31)	$\boxed{a_2\colon \Diamond_R\neg a_3}$	$28 \leadsto^{@} 31$
32)	$a_4\colon \Diamond_R\downarrow y.a_4\colon \Diamond_R\neg y$	$30 \leadsto^{\downarrow} 32$
33)	$a_2\colon \Diamond_R a_5$	$31 \leadsto^{\Diamond} 33$
34)	$a_5\colon \neg a_3$	$31 \leadsto^{\Diamond} 34$
35)	$a_4\colon \Box_{R^-}p$	$(6,24) \leadsto^{\mathsf{A}} 35$
36)	$a_3\colon \Box_{R^-}p$	$(6,22) \leadsto^{\mathsf{A}} 36$
37)	$a_2\colon \Box_{R^-}p$	$(6,15) \leadsto^{\mathsf{A}} 37$
38)	$a_1\colon p$	$(35,24) \leadsto^{\Box} 38$
39)	$a_2\colon p$	$(36,22) \leadsto^{\Box} 39$
40)	$a_4\colon \Diamond_R a_6$	$32 \leadsto^{\Diamond} 40$
41)	$a_6\colon \downarrow y.a_4\colon \Diamond_R\neg y$	$32 \leadsto^{\Diamond} 41$
42)	$a_6\colon \Box_{R^-}p$	$(6,40) \leadsto^{\mathsf{A}} 42$
43)	$a_4\colon p$	$(42,40) \leadsto^{\Box} 43$

$F = \Diamond_R \top \wedge \mathsf{A}\Box_{R^-} p \wedge \Box_R G$, where $G = \downarrow x.\Diamond_R \downarrow y.x{:}\,\Diamond_R \neg y$ and R is transitive

0)	$a_0{:}\, F$	
1)	$\mathrm{Trans}(R)$	
2)	$R \sqsubseteq R$	
3)	$a_0{:}\, (\Diamond_R \top \wedge \mathsf{A}\Box_{R^-} p)$	$0 \rightsquigarrow^{\wedge} 3$
4)	$a_0{:}\, \Box_R G$	$0 \rightsquigarrow^{\wedge} 4$
5)	$a_0{:}\, \Diamond_R \top$	$3 \rightsquigarrow^{\wedge} 5$
6)	$a_0{:}\, \mathsf{A}\Box_{R^-} p$	$3 \rightsquigarrow^{\wedge} 6$
7)	$a_0{:}\, \Diamond_R a_1$	$5 \rightsquigarrow^{\Diamond} 7$
8)	$a_1{:}\, \top$	$5 \rightsquigarrow^{\Diamond} 8$
9)	$a_1{:}\, \Box_{R^-} p$	$(6,7) \rightsquigarrow^{\mathsf{A}} 9$
10)	$a_0{:}\, \Box_{R^-} p$	$(6,0) \rightsquigarrow^{\mathsf{A}} 10$
11)	$a_1{:}\, \Box_R G$	$(4,7,1,2) \rightsquigarrow^{\mathrm{Trans}} 11$
12)	$a_1{:}\, G$	$(4,7) \rightsquigarrow^{\Box} 12$
13)	$a_0{:}\, p$	$(9,7) \rightsquigarrow^{\Box} 13$
14)	$a_1{:}\, \Diamond_R \downarrow y.a_1{:}\, \Diamond_R \neg y$	$12 \rightsquigarrow^{\downarrow} 14$
15)	$a_1{:}\, \Diamond_R a_2$	$14 \rightsquigarrow^{\Diamond} 15$
16)	$a_2{:}\, \downarrow y.a_1{:}\, \Diamond_R \neg y$	$14 \rightsquigarrow^{\Diamond} 16$
17)	$a_2{:}\, \Box_R G$	$(11,15,1,2) \rightsquigarrow^{\mathrm{Trans}} 17$
18)	$a_2{:}\, G$	$(11,15) \rightsquigarrow^{\Box} 18$
19)	$a_2{:}\, a_1{:}\, \Diamond_R \neg a_2$	$16 \rightsquigarrow^{\downarrow} 19$
20)	$a_2{:}\, \Diamond_R \downarrow y.a_2{:}\, \Diamond_R \neg y$	$18 \rightsquigarrow^{\downarrow} 20$
21)	$a_1{:}\, \Diamond_R \neg a_2$	$19 \rightsquigarrow^{@} 21$

22)	$a_2{:}\, \Diamond_R a_3$	$20 \rightsquigarrow^{\Diamond} 22$
23)	$a_3{:}\, \downarrow y.a_2{:}\, \Diamond_R \neg y$	$20 \rightsquigarrow^{\Diamond} 23$
24)	$a_1{:}\, \Diamond_R a_4$	$21 \rightsquigarrow^{\Diamond} 24$
25)	$a_4{:}\, \neg a_2$	$21 \rightsquigarrow^{\Diamond} 25$
26)	$a_3{:}\, \Box_R G$	$(17,22,1,2) \rightsquigarrow^{\mathrm{Trans}} 26$
27)	$a_3{:}\, G$	$(17,22) \rightsquigarrow^{\Box} 27$
28)	$a_3{:}\, a_2{:}\, \Diamond_R \neg a_3$	$23 \rightsquigarrow^{\downarrow} 28$
29)	$a_4{:}\, \Box_R G$	$(11,24,1,2) \rightsquigarrow^{\mathrm{Trans}} 29$
30)	$a_4{:}\, G$	$(11,24) \rightsquigarrow^{\Box} 30$
31)	$a_2{:}\, \Diamond_R \neg a_3$	$28 \rightsquigarrow^{@} 31$
32)	$a_4{:}\, \Diamond_R \downarrow y.a_4{:}\, \Diamond_R \neg y$	$30 \rightsquigarrow^{\downarrow} 32$
33)	$a_2{:}\, \Diamond_R a_5$	$31 \rightsquigarrow^{\Diamond} 33$
34)	$a_5{:}\, \neg a_3$	$31 \rightsquigarrow^{\Diamond} 34$
35)	$a_4{:}\, \Box_{R^-} p$	$(6,24) \rightsquigarrow^{\mathsf{A}} 35$
36)	$a_3{:}\, \Box_{R^-} p$	$(6,22) \rightsquigarrow^{\mathsf{A}} 36$
37)	$a_2{:}\, \Box_{R^-} p$	$(6,15) \rightsquigarrow^{\mathsf{A}} 37$
38)	$a_1{:}\, p$	$(35,24) \rightsquigarrow^{\Box} 38$
39)	$a_2{:}\, p$	$(36,22) \rightsquigarrow^{\Box} 39$
40)	$a_4{:}\, \Diamond_R a_6$	$32 \rightsquigarrow^{\Diamond} 40$
41)	$a_6{:}\, \downarrow y.a_4{:}\, \Diamond_R \neg y$	$32 \rightsquigarrow^{\Diamond} 41$
42)	$a_6{:}\, \Box_{R^-} p$	$(6,40) \rightsquigarrow^{\mathsf{A}} 42$
43)	$a_4{:}\, p$	$(42,40) \rightsquigarrow^{\Box} 43$

The offspring relation

Root nodes: 0–6 and 10.

$5 \prec_{\mathcal{B}} \{7 - 9, 11 - 14\}$
$20 \prec_{\mathcal{B}} \{22, 23, 26 - 28, 31, 36, 39\}$
$31 \prec_{\mathcal{B}} \{33, 34\}$

$14 \prec_{\mathcal{B}} \{15 - 21, 37\}$
$21 \prec_{\mathcal{B}} \{24, 25, 29, 30, 32, 35, 38\}$
$32 \prec_{\mathcal{B}} \{40 - 43\}$

For instance:

- $$\left.\begin{array}{r} (4,7,1,2) \leadsto^{\text{Trans}} 11 \\ (4,7) \leadsto^{\Box} 12 \\ (9,7) \leadsto^{\Box} 13 \end{array}\right\} \Longrightarrow \quad 7, 11, 12, 13 \text{ are siblings.}$$
- $(6,7) \leadsto^{A} 9$, because 7 is the first non-phantom node where the focused nominal a_1 occurs $\Longrightarrow$ 7 and 9 are siblings.
- $(6,22) \leadsto^{A} 36$, because 22 the first non-phantom node where the focused nominal a_3 occurs when the rule is applied (even if afterwards 22 becomes a phantom) $\Longrightarrow$ 22 and 36 are siblings.

Comments

Notation: $\mathcal{B}_n$ is the branch segment up to node n included.

- In $\mathcal{B}_{43}$, both 20 and 32 are blocked by 14, and all their descendants $(22, 23, 26–28, 31, 33, 34, 36, 39–43)$ are phantoms in $\mathcal{B}_{43}$.
- 20 is blocked by 14 only in $\mathcal{B}_{37}$ and from $\mathcal{B}_{39}$ onwards.
- 32 is blocked by 14 also in $\mathcal{B}_{35}$ and $\mathcal{B}_{36}$.
- 32 is blocked by 20 in $\mathcal{B}_i$ for $32 \leq i \leq 34$ and $i = 38$ (though 20 is not an ancestor of 32).
- 40 is not a phantom in $\mathcal{B}_{42}$, so that it can be used as the minor premiss of the application of the $\Box$ rule producing 43. As soon as 43 is added to the branch, 40 becomes a phantom.
- 31 is never directly blocked: in order to be blocked by 21, a_1, a_2 and a_3 must be compatible. But when a_1 and a_2 are compatible, 20 is blocked, and 31 is a phantom.

The branch is complete

No further expansion are possible without violating the restrictions on blocked nodes.

In particular, in the whole branch:

- the A rule cannot focus on a_5, which only occurs in phantom nodes.
- Though nodes 36 and 42, obtained by applications of the A rule, are phantoms, such a rule cannot focus again on a_3 and a_6, which only occur in phantom nodes.
- Though 26 and 27 are phantoms, the Trans and $\Box$ rules cannot use again 22 as a minor premiss, since it is a phantom too.
- Similarly, the other phantom nodes labelled by relational formulae cannot be used as minor premisses. For instance, 40 cannot be used as the minor premiss of an application of the $\Box$ rule, paired with 29.

Remark: 20 and 32 are blocked by 14
$\Longrightarrow$ intuitively, a_2 and a_4 behave "like" a_1.

However, though a_2 and a_4 are compatible, the presence of node 25 does not allow to identify the states they denote.